XIV. The Bakerian Lecture.—*On the Transparency of the Atmosphere and the Law of Extinction of the Solar Rays in passing through it. By* James D. Forbes, *Esq., F.R.S., Sec. R.S. Ed., Corresponding Member of the Royal Institute of France, and Professor of Natural Philosophy in the University of Edinburgh.*

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Contents.

1. THE experiments which will chiefly be analysed in this paper were made nearly ten years since. I have been deterred from drawing the conclusions from them which they warrant, partly by the great labour of the necessary reductions, and partly by peculiar and inherent difficulties which this intricate subject presents. For the computations I am much indebted to the perseverance and care of Mr. John Broun (now magnetical assistant to Sir Thomas Brisbane), who has made most of them under my eye, and also to Mr. James Stark. For much which yet remains obscure and uncertain in my conclusions, I anticipate the indulgence of those best acquainted with the uncertainties under which the subject of absorption, whether of light or heat, is still veiled, and with the little advance which has been made in the particular branch which we have to consider, namely the law of extinction of solar light and heat in the atmosphere.

2. Permanently enclosed as we are within an imperfectly transparent shell which separates us from the realms of space, a knowledge of the various properties of the atmosphere, especially as regards light and heat, is peculiarly important in the resolution of many cosmical problems. We cannot at will place ourselves, as it were, in a point in space, until we can eliminate the effects of this transmission. Hence the great importance of the subject of astronomical refractions, one nearly allied to the present, and which has exercised, from the time of Newton to that of Ivory, the happiest skill of some of the most eminent analysts and natural philosophers. The difficulties of the doctrine of astronomical refractions it has in common with that of the extinction of light, and to these are superadded many more, owing to the incomparably inferior methods which we have of measuring both light and heat compared to the measure of angular quantities.

SECTION I.—*Qualities of Rays.*

3. Not to incur a charge of vagueness, to which our remarks may be thought liable when we speak of light and heat together, I would first notice the difficulty under which we labour in this respect. Our instruments are so imperfect that it is difficult to say what particular effect we are truly measuring at a given time. The solar rays contain light and heat; and each of these results of radiations varies, first, in respect of intensity, and secondly, in respect of quality. We may *seem* to perceive a change in the first of these effects when it is only the second which has varied. Light may be more or less brilliant, but it may also be red or yellow. Heat may be more or less intense, but it also changes its quality in a manner similar to the effect of coloration for light. A person who could distinguish only yellow rays of light, would feel as if plunged in darkness when the red ray of the spectrum was directed on his eye. Our instruments for the measurement of heat possess undoubtedly something like the quality now supposed to exist in a natural eye. They may indicate, not the whole intensity of the incident heat, but only the intensity of that modification of it, to which the measurer applied is exclusively or peculiarly sensitive. As we employ a black, or a blue, or a white thermometer to measure the force of the solar rays, we shall have indications not only *absolutely* different, but *relatively* different under different circumstances. Interpose, for instance, a plate of glass, and though the same reduced quantity of heat falls upon all the three instruments, the black thermometer will sink less than the white one would do*. We cannot tell but that the atmosphere acts as the plate of glass does, and therefore, that the indication of the opacity of the atmosphere in respect to the heating rays is only true as regards a certain class of heating rays: nay, it is almost demonstrated that such is the case; and that consequently we must use "law of extinction," "transparency of the atmosphere," and such terms, with especial and exclusive reference to the class of effects which our instrument is capable of measuring.

4. The direct quantitative measurement of light has not yet been satisfactorily accomplished; and of the indirect methods, some depend upon the faculty of the eye in comparing illuminated surfaces, and others upon the thermometric effects which the luminous calorific rays produce. We can by no means conclude that these two methods, so dissimilar, of estimating the loss of solar light in its transit through the atmosphere, ought to give identical results. The one was practised by BOUGUER, the other by LAMBERT; and it is to the latter class alone that the experiments to be described in this paper belong: but before detailing them, it may be well to glance historically at the more important parts of the problem.

SECTION II.—*History of the Inquiry.*

5. Sir ISAAC NEWTON, in the third book of his Optics, speaks of the opacity or imperfect transparency of bodies as arising from the multiplied reflections of light

* POWELL, Philosophical Transactions, 1825.

in their interior, owing to a want of perfect homogeneity; but he does not seem to have pushed the consideration of the matter much further, at least so far as I recollect of the history of optics of his time. It is to BOUGUER that we owe the first careful consideration of the varying intensities of absorbed and reflected light, and a special application of it to the case before us, the transparency of the atmosphere. Father FRANÇOIS MARIE had, indeed, in 1700, described an instrument intended to measure the intensity of light, but he failed grievously in his proposed application of it*. It consisted of a series of plates of glass, or increasing thicknesses of water, to be interposed between the eye and the object until the light should be wholly stopped. But the writer proceeded on the inaccurate idea, that each successive plate, or increment of thickness, would stop an equal proportion of the *original* light instead of an equal share of the light *incident* upon it. The former, being an arithmetical progression, would produce a speedy and complete extinction; the latter would give a continually diminishing geometrical proportion, which would approach slowly and indefinitely to zero.

6. This last analogy BOUGUER perceived and proved in a tract published in 1729, which was only the precursor to his great work, published after his death, under the title of "Traité d'Optique sur la Gradation de la Lumière"†. He shows that, from the geometrical law of extinction just alluded to, the remaining intensity of light, after having passed through any thickness of a *uniformly dim* medium, may be represented by the ordinates of a logarithmic curve, the abscissæ denoting the thickness. This property he ingeniously applies with considerable mathematical skill to a variety of cases. The chief of these is to the transparency of the atmosphere.

7. MAIRAN had already shown‡ that the varying thickness of the atmosphere, traversed by rays from the heavenly bodies at different altitudes, during their diurnal course, produces a continual variation in their apparent brightness; and he anticipated the possibility of deducing the total loss in *one* transit by comparing the losses due to different thicknesses. But he was ignorant of the logarithmic law discovered by BOUGUER; for he supposed the losses of light proportional to the lengths of the paths traversed§. The latter gave the theoretical solution of the problem, and applied it to practice. It being inferred from what has been already stated, that the intensities are in a geometrical progression when the thicknesses vary arithmetically, it follows that the thickness traversed, of a homogeneous medium, is proportional to the difference of the logarithms of the incident and transmitted light, or what comes to the same thing, it is proportional to the logarithm of their ratio. Thus, if for atmospheric thicknesses x_1 and x_2, the transmitted light be v_1 and v_2; also, if V be the intensity of light exterior to the atmosphere, and m a constant

$$\log \frac{V}{v_1} = m\, x_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1.)$$

* Nouvelle Découverte sur la Lumière pour en mesurer et compter les degrés. Cited in MONTUCLA, Histoire des Mathématiques, iii. 538, where there is a good sketch of the history of photometry.

† 4to, Paris, 1760. ‡ Mémoires de l'Academie, 1721. § Mém. p. 14.

$$\log \frac{V}{v_2} = m\, x_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.)$$

Subtracting

$$\log \frac{v_2}{v_1} = m\,(x_1 - x_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.)$$

where everything is known except m, which is thus determined, and which substituted in eq. (1) gives the value of V the unknown intensity exterior to the atmosphere in terms of the same unit as v_1 and v_2. For eq. (1) may be written

$$\log V - \log v_1 = m\, x_1$$

and

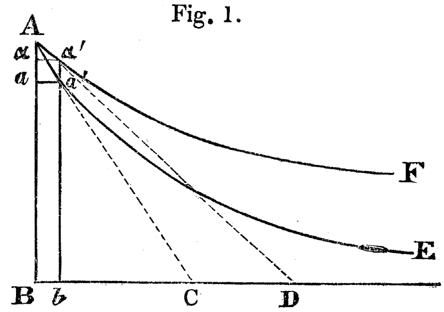
$$\log V = \log v_1 + m\, x_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.)$$

8. This was the principle of BOUGUER'S solution; the values of v_1 and v_2 were determined by comparing the intensity of moonlight for different elevations. Thus at elevations of 19° 16′ and of 66° 11′ the intensities were as three to two when compared by means of the light of wax candles at variable distances*. The values of x_1 and x_2 were determined from the supposed constitution of the atmosphere in a manner we shall afterwards make known.

9. The conclusion at which BOUGUER on the whole arrived was, that the light of the sun and stars would, by a *simple vertical* transmission through the atmosphere, be reduced to 0·8123 of its original brightness†.

10. It is evident from the preceding investigation, that as the extinction of the ray depends on the rate of inclination of the logarithmic curve to its axis at any point, in different homogeneous substances, the intensity of the incident light being the same, the decreasing ratio of the light is inversely as the constant subtangent. Thus, let A B represent the light incident on a plate whose thickness is reckoned in the direction B D, and let A a be the decrement due to the passage of the ray through the thickness B b of any medium: let A α be the smaller decrement due to the action of a more transparent medium: then

Fig. 1.



$$A\,a : A\,B = B\,b : B\,C$$

and

$$A\,\alpha : A\,B = B\,b : B\,D,$$

whence

$$A\,a : A\,\alpha = B\,D : B\,C,$$

or the decrement is inversely as the subtangent of each curve.

* Four wax candles forty-one feet off equalled the intensity of moonlight with an elevation of 66° 11′, and the same number at a distance of fifty feet corresponded to an elevation of 19° 16′. The intensities are as the squares of the distances inversely, or as 2500 to 1681, or nearly as 3 : 2.

† Traité d'Optique, p. 332.

11. The definition of transparency, according to BOUGUER, is *the reciprocal of the thickness of the medium required to diminish the incident light in a given ratio.*

12. In the same year with the posthumous publication of BOUGUER, appeared the "Photometria" of LAMBERT; a work of great ingenuity and labour, based in many respects upon BOUGUER's earliest investigations already mentioned, to which the author fails not to give due credit. This work, owing to some circumstance, has always been very scarce: PRIESTLEY appears never to have been able to get a copy, and now it is one of the rarest of modern scientific works. It was printed under the title of "Photometria, sive de Mensura et Gradibus Luminis, Colorum, et Umbræ. Augustæ Vindelicorum, 1760, 8vo, pp. 547"*. In the first chapter of the fifth part, the author treats of the transparency of the atmosphere. His methods of finding the thickness of air traversed at different altitudes are less accurate approximations than that of BOUGUER. His method of observing the intensity of radiation was very different, as well as the result. He observed the degree at which a thermometer lying in the sun rose above one in the shade, and he took this difference as a measure of solar radiation. The experiments briefly cited in the "Photometria"† are fully detailed in his "Pyrometria," published in quarto in 1779‡. LAMBERT's observations on the intensity of the solar rays were made during a considerable part of the year. The following made at Coire on the 17th of May 1756, he selects as the best: the height of the barometer was twenty-six French inches.

Sun's Altitude.	REAUM. Excess in the Sun.
60°	15°·8
50	14·6
40	12·8
30	10·0.

Hence, by a procedure the same as has been above explained, he deduces the light transmitted through the atmosphere in a vertical direction, which he estimates at 0·5889 of the incident rays, being less than $\frac{3}{5}$ instead of more than $\frac{4}{5}$, as BOUGUER had supposed.

13. LAMBERT's work on Pyrometry is further remarkable for the clear description of several methods and results which have in later times been rediscovered or successfully applied. He was the first to apply calculation to the flow of heat through solid bodies§: he discovered the law of the intensity of radiant heat proportional to the sine of the angle made with the heated surface‖. He was aware that the true measure of the cause of radiant heat was the *velocity* of heating resulting from it¶. He applied this method to the determination of the permeability of successive plates of glass to the solar rays, constructing for each experiment the curve of rise of tem-

* The analysis of this work in Montucla is imperfect. † page 396. § 886.
‡ page 158. § 283. § Pyr. § 327. ‖ Pyr. § 319. ¶ § 270.

perature in terms of the time elapsed, from which he deduced the velocity of heating under similar circumstances. The result of these experiments is remarkable as anticipating the law usually attributed to De la Roche, that the facility of transmission through successive plates of glass continually increases with the number already passed through*. He thus finds

		Loss.
Incident heat	100	
Through one plate of glass . . .	84	16
Through two plates of glass . .	69	15
Through three plates of glass . .	59	10.

We shall have occasion to return to this important experiment.

14. In 1774 De Saussure employed an instrument which he called a Heliothermometer, for measuring the force of the solar rays, particularly upon the top of mountains. It consisted of a wooden box lined with thick pieces of blackened cork, and covered with three separate superimposed glass plates which admitted the solar rays to a thermometer placed in contact with the cork: in this instrument the mercury rose to 70° Reaumur†.

15. Sir John Leslie, in his Essay on Heat‡, proposed a modification of his differential thermometer with one bulb blackened, and the other clear, as a photometer; since the excess of effect on the dark ball appearing only when heat is accompanied by light, might, he thought, be considered as a measure of the light. There is, however, a twofold objection to consider this instrument as exact in its indications. 1st. The quality of affecting dark surfaces more than pale ones, is a quality of heat often accompanying, but not inseparable from light. This has already been proved from Professor Powell's experiments, which have been further confirmed by M. Melloni, who finds, for instance, that a black and a white surface absorb lamp heat in the ratio of 100 to 80·5; and if the heat be transmitted through rock salt, the ratio is unchanged; but if alum be used instead of salt, though equally permeable to light, the ratio now becomes 100 : 42·9. But, secondly, an objection clearly foreseen by Lambert, is applicable to all measures of a radiant source by the *statical* effect on a thermometer. The condition of a body remaining at a higher temperature than the surrounding medium is this:—that it shall receive in a given time as much heat as it parts with: the more it receives the higher must be the temperature to which it must be raised in order to part (by the law of cooling) with an equal amount. The measure of intensity depending upon the stationary heat of the thermometer, clearly supposes that the cooling proceeds according to a constant law in all the experiments which are to be compared§.

* Pyrometrie, § 282. † Voyages dans les Alpes, § 932. ‡ 8vo, Lond. 1804.

§ The form which Lambert gave to his formula was this,—

$$dy = n\,dt - \frac{y\,dt}{S}$$

where y is the excess of temperature marked by the thermometer, n the heat communicated directly in unit of

16. The former of these objections applies to all thermometric instruments considered as *light* measurers; but the latter has been ingeniously got over by Sir JOHN HERSCHEL, in a way more convenient in practice than that used by LAMBERT.

17. HERSCHEL'S actinometer consists of a thermometer with a large cylindric bulb, containing a deep-blue fluid (the ammonio-sulphate of copper), and inclosed in a wooden case, blackened interiorly and covered with a piece of thick plate glass. The capacity of the bulb may be caused to vary, by screwing in or out a plunger which enters parallel to the axis of the cylinder, and the use of which is to retain the top of the column of fluid within the range of the tube, which is connected with the cylinder as in the common thermometer, and which it would otherwise be liable to exceed, owing to the great variations of temperature to which it is exposed. The *velocity* of heating during exposure to the sun is ascertained by limiting the exposure to *one minute*, during which the rise of the liquid is accurately observed. But since during this minute, the rise was not that due to the solar influence alone, but to the direct solar influence *plus* or *minus* all the cooling or heating influences simultaneously acting on the actinometer, these indirect influences are ascertained and allowed for by exposing the instrument for *one minute* behind a screen, which merely stops the solar rays, but allows all other actions to go forward. If the instrumental readings *fall* during the shade observation (owing to the coolness of the atmosphere and the high temperature of the liquid), it is plain that the solar action was not only to *raise* but to *maintain* the temperature, and that the fall during the shade observation must be added to the rise during the sun observations to give the effect due to the sun. On the other hand, if the temperature continue to rise during the shade observation (which may be due to indirectly reflected heat, or to the communication of heat from the parts of the instrument), it is plain that the rise in the sun was not wholly due to the immediate solar influence, and therefore that the rise in the shade must be subtracted from it.

18. This instrument gives very constant and satisfactory results: it is the one with which the following observations were entirely made; examples of the mode of using it will therefore be given when I come to describe my own experiments. In the mean time I may refer for the first description of the actinometer to the Edinburgh Journal of Science for 1825*; and for a very full account of it, and the method of using it by Sir JOHN HERSCHEL himself, in the 'Instructions' lately published by the Royal Society†.

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time, t the time, and S the subtangent of the logarithmic curve which expresses the Newtonian law of cooling, having the excess of temperature above the medium for its ordinate, and the time for its abscissa. This subtangent varies (as he observes § 282) with the state of the atmosphere. It is very remarkable that LESLIE himself pointed out in his very earliest published composition (an Essay written in 1792 or 1793, but first printed in THOMSON'S Annals of Philosophy in 1819, vol. xv. p. 7.), that "the *initial change*" (or rate of heating of a black surface exposed to the sun) "on the thermometer, is in every case *the only certain and accurate measure* of the communication of heat":—a principle which, however, he practically abandoned, as we have seen.

* Vol. iii. p. 107.

† p. 58.

19. The scale of the actinometer is an arbitrary one obtained by the direct comparison of one instrument with another; a method which, as we shall see, admits of great accuracy. Sir JOHN HERSCHEL has, indeed, proposed to convert his degrees into "actines," each of which represents "that intensity of solar radiation, which at a vertical incidence, supposing it wholly absorbed, would suffice to melt one-millionth part of a metre in thickness from a sheet of ice horizontally exposed to its action, per minute of mean solar time*". It may be apprehended, however, that an arbitrary comparison will always be found more available in practice. The scale of LESLIE's photometer should also be considered as arbitrary; the method of graduation by conversion into hygrometric degrees being wholly inaccurate.

20. LESLIE, from his experiments made on the sun's force at different elevations, concluded that *one-fourth* of the solar rays are absorbed during a vertical transmission through the atmosphere in clear weather at Edinburgh†. His experiments were made under the revolving dome of an observatory‡. Thus the indirect heating and cooling influences were in some degree avoided. These influences are of a very material kind, and prove the impossibility of obtaining even an approximation to useful results without allowing for them. The photometer of LESLIE indicates as much effect (according to Professor KÄMTZ) due to the light reflected from the atmosphere as to the direct solar influence; certainly a most startling result, but one entirely confirmed by my own observations. The part exclusively due to solar influence being taken by M. KÄMTZ, he finds from BOUGUER's formula an extinction of no less than thirty per cent. of the solar rays in reaching the summit of the Faulhorn, by a vertical transit through the atmosphere, the pressure of which amounts there to only about twenty-one English inches§.

21. M. POUILLET, of Paris, described some years ago an apparatus for measuring solar radiation, in which the errors of other statical contrivances were in a great measure avoided, by enclosing the thermometer in an envelope maintained at 0° cent., with the exception of a small hole which exactly admitted the direct rays from the solar disk. Since that time he has adopted HERSCHEL's dynamical method, which he has applied to a modification of the actinometer, which he terms a *pyrheliometer*; reserving (rather unfortunately, I think) the term actinometer, which was already so fitly appropriated, to a separate apparatus for measuring nocturnal radiation. These instruments and their applications are described in an ingenious and interesting memoir read to the Academy of Sciences, 9th July 1838, printed in the Comptes Rendus, and privately circulated under the title of "Mémoire sur la Chaleur Solaire sur les Pouvoirs Rayonnants et Absorbants de l' Air Atmosphérique, et sur la Température de l'Espace."

22. The pyrheliometer is composed of a thin metallic chamber containing water,

* Royal Society's Instructions, p. 65.

† Article 'Climate,' Encyclopædia Britannica.

‡ See my Supplementary Report on Meteorology in the British Association Report for 1840, p. 63.

§ Lehrbuch der Meteorologie, iii. 14.

blackened externally, and exposed to solar radiation, having a thermometer plunged into it, which ascertains the rate of heating or cooling of the fluid in the chamber. It is observed by sun and shade alternating series, after the manner invented by HERSCHEL, and the whole instrument is only a less perfect modification of the actinometer under a form slightly different. M. POUILLET, from observations at Paris, finds the absorption of solar heat to follow very rigorously the law of BOUGUER,—namely, that the mass of air traversed varies as the logarithm of the ratio of the intensity observed to the constant intensity beyond the atmosphere. He gives it the form*

$$t = \mathrm{A} p^{\varepsilon};$$

where t is the observed intensity, A is what he calls the solar constant (*i. e.* the intensity beyond the atmosphere), p the atmospheric constant which determines the opacity, and ε the mass of air to be traversed.

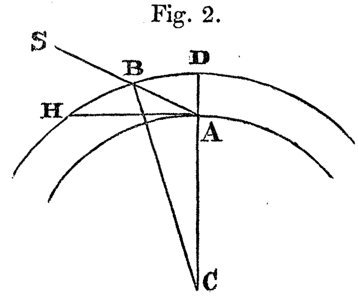
23. That this comes under BOUGUER's form is easily seen by taking the logarithms of both sides :—

$$\log \frac{t}{\mathrm{A}} = \varepsilon \log p,$$

where $\log p$ is a constant. From his various observations M. POUILLET concludes that the loss by vertical transmission is sometimes as low as eighteen per cent., and seldom higher than twenty-five per cent. when the sky appears pure.

SECTION III.—*On the Mass of Atmospheric Air traversed by rays with varying obliquities.*

Fig. 2.



24. It follows, from the simplest geometrical considerations, that if the strata of air be ranged concentrically, and if the thickness of the atmosphere A D be small compared to the radius of the globe C A, the length of path A B of the rays of light will vary nearly as the secant of the zenith distance, except near the horizon. It is, of course, supposed (which is the fact) that the curvature of the path A B is so inconsiderable as not materially to affect its length. Within the limits between which the above approximation may be accepted as correct (that is, for altitudes above 15° for most purposes), it is plain that the law of densities at different heights is left out of account; for the hypothesis amounts to considering the strata as *flat*, and therefore as all cut by the ray at the same angle.

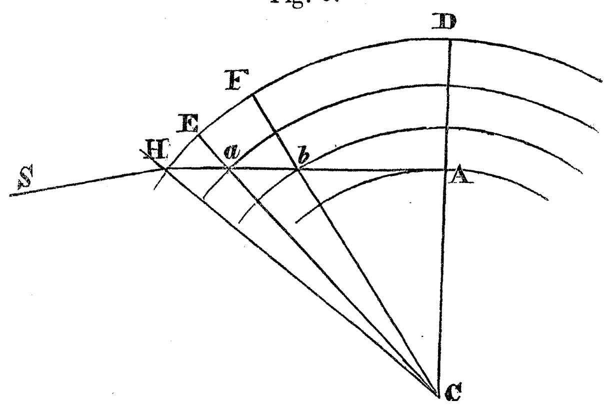
25. Near the horizon, however, this law can give no approximation, for there the secant would give an infinite thickness traversed, whilst the curvature of the globe and atmosphere limits the path to the length of the line A H. If we would, therefore, ascertain the length of path, we must ascertain the length of A H; and so

* Mémoire, p. 6.

for all low elevations. This infers a knowledge of the height of the atmosphere; but more than this, it requires that we should be able to ascertain the density of each particular stratum of air, for each stratum is traversed at a different obliquity, and the quantity of matter which each stratum opposes to the passage of the ray is evidently proportional to the density of the stratum, and the length of path in the stratum. The mass of air penetrated will be the integral of all such elementary parts.

26. Supposing the atmosphere divided into three strata of equal thickness but differing in density, it is evident that the horizontal ray H A will traverse the com-

Fig. 3.



paratively short space H a of the highest, the longer space a b of the middle stratum, and by far the longest course b A in the lowest (the curvature of the path being small). Now as the lowest stratum is also the densest, it follows that on both these accounts the stratum in which we are placed acts most energetically, and hence the thickness traversed by a horizontal ray may be represented by a highly converging series upon almost every physically-possible hypothesis of density.

27. This figure also shows that, supposing the strata numerous and thin, the thickness of each stratum traversed will be equal to its vertical thickness multiplied into the secant of the angle of incidence of the transmitted ray.

28. LAMBERT contented himself with finding in the first place in fig. 2. the length of the line A H or A B from simple geometrical considerations, which he gives in these terms*,

$$\cos\gamma + x = \sqrt{\cos^2\gamma + 2y + y^2};$$

where γ is the zenith distance, y is the height of the particular stratum D B H (the earth's radius being $= 1$), and x is the length of the path A B or A H. He then differentiates the expression in respect of x and y, and multiplying the element of the path thus found by the density of the stratum (variable with y according to some law to be assumed), he expands the quantity to be integrated in a series, of which, however, he has not attempted to find the exact value, but stops at the first term which is proportional to the secant of the zenith distance, as we have seen. The only person

* Photometria, p. 393.

who, so far as I know, has pushed the approximation *practically* to the second term, which has the form

$$B \tan^3 \gamma . \sec \gamma,$$

is Professor KÄMTZ*. M. POUILLET has simply adopted the first formula above given, which, as it supposes the density constant, will make the mass of air traversed appear too small, producing in his results an accidental compensation of errors to which we shall afterwards allude.

29. LAMBERT's approximations are entirely of a tentative kind, that is, deduced *à posteriori* by ascertaining from a number of observations corresponding to the number of unknown coefficients, the successive terms of a series or points of an interpolating curve.

30. This method is objectionable in this respect,—that it proceeds upon the assumption of the uniform opacity of equal successive masses of air, upon which alone the logarithmic form of the law of absorption is correct. Since our object ought to be to verify this law, we must have a direct method of ascertaining the mass of air traversed by a ray at different elevations, which can only be founded on an *approximate* knowledge of the constitution of the atmosphere.

31. BOUGUER had previously solved the problem in a direct manner, though by approximation, which is indeed the only method it admits of. Assuming the logarithmic law of densities and heights in the atmosphere (as in the common barometric formula), and supposing the temperature constant, he obtained a converging series for the atmospheric mass traversed by a horizontal ray, and also at different elevations†. This he expressed in thicknesses of air of the common density at the earth's surface: assuming the height of the equiponderant column of a uniform atmosphere at 3911 toises, he finds the mass traversed at 45° of elevation to be 5530 toises, and at the horizon 138,823 toises. Notwithstanding that he says that the approximation was not pushed very far, we shall show presently that BOUGUER's determination corresponds well with that obtained by the most recent methods.

32. LAPLACE has considered the subject of the extinction of light by the atmosphere in the third chapter of the tenth book of the *Mécanique Céleste*. He has there ingeniously established an analogy between the amount of astronomical refraction and the mass of air traversed by a ray in any direction. By this means, the ample knowledge which we at present possess respecting astronomical refractions becomes immediately applicable to the subject before us.

33. I may remark, however, in passing, that in inquiring into the law of the extinction of light by the atmosphere, we would do well to avoid much use of observations near the horizon, the opacity of the vapours in the atmosphere introducing a variable and important element not recognizable with any accuracy by common meteorological observations. Any law of extinction will, therefore, be better determined from multiplied observations at elevations above 15°, than by those nearer the horizon, which are liable to more than all the objections to astronomical observations made

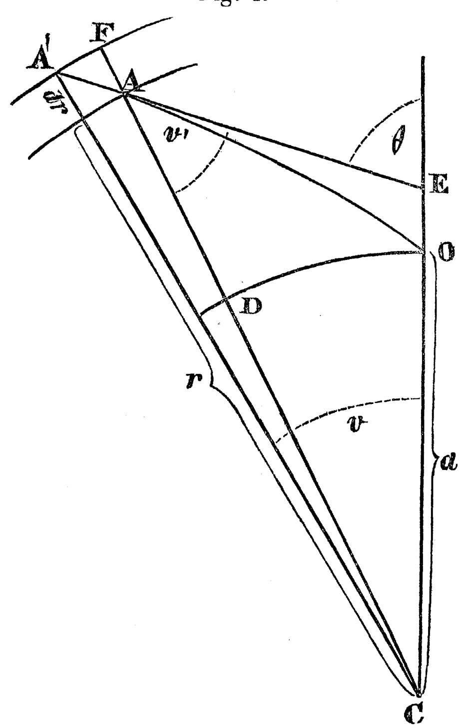
* Météorologie, III. 13.

† Traité d'Optique, p. 331.

under similar circumstances. For smaller zenith distances it has already been observed that LAMBERT's simple law of the secant gives an approximation quite within the limits of error of the observations to be compared. I have, therefore, adopted it uniformly in my reductions, partly for the sake of simplicity, partly on account of a doubt I at one time entertained respecting the admissibility of one of LAPLACE's approximations,—a doubt which was removed by consultation with my colleague, Professor KELLAND. I think it may be useful, however, to give a short proof of LAPLACE's method, on account of its accuracy and simplicity, in which everything belonging merely to the subject of refraction shall be omitted, and only what is essential to the law of extinction considered. In doing this I shall avail myself of the valuable assistance afforded by BOWDITCH's notes to his translation of the *Mécanique Céleste*.

34. i. *To find the differential equation of the Intensity of transmitted Light.*—Let O D. (fig. 4.) be a portion of the earth's surface, C its centre, A′ A O the path of a ray of

Fig. 4.



light; $CA = r$ the radius of any concentric stratum of the atmosphere, whose thickness $AF = dr$. Let the angle $A'AF = EAC = v'$, which denotes the angle with the radius made by the ray in passing through the stratum under consideration. Then the thickness of the stratum traversed will be $AA' = dr \sec v'$; and if the density of the air in the stratum be ϱ, the mass of air passed through varies as $\varrho\, dr \sec v'$.

35. Let ε represent the intensity of the light just entering the stratum A′ F, then it will suffer a decrement $-d\varepsilon$ in passing through the stratum A′ A, bearing a constant proportion to the brightness of the incident beam and to the resistance which it has to encounter (supposing the opacity of a medium to depend solely on the number of

material particles which it contains, without reference to their distribution). Consequently

$$d\varepsilon = -Q.\varepsilon.\varrho\, dr.\sec v' \quad \text{. . . . .} \quad (1.)$$

Q being a constant quantity.

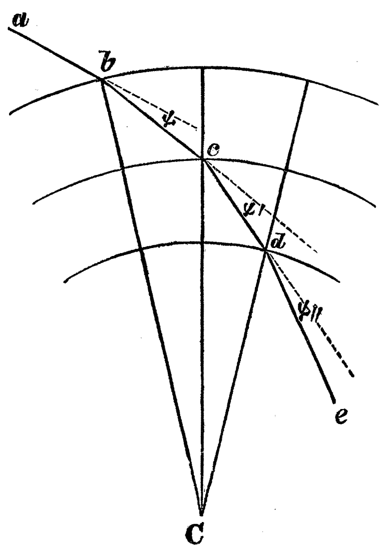
36. ii. *To find the differential of refraction.*—In fig. 4. θ is the angle formed with the zenith by any element A′ A of the path of a ray A′ A O refracted in the atmosphere. Hence the variation of θ or $d\theta$ is the differential of refraction. Now

$$\theta = v + v' \quad \text{. . . . .} \quad (2.)$$

$$d\theta = dv + dv' \quad \text{. . . . .} \quad (3.)$$

37. iii. When light is refracted at successive concentric surfaces of varying density, the flexure of the ray at each is the differential element of refraction. Thus in fig. 5. it is evident that the course of a ray being represented by the polygon *a b c d e*, the

Fig. 5.



flexure at the common surface of each successive stratum ψ, ψ', ψ'', &c., due to the inequality of the angles of incidence and refraction, measures the actual deviation of the ray from a rectilinear course*. Hence

$$\text{refraction} = \psi + \psi' + \psi'' + \&c.,$$

and

$$\delta\theta = \psi \quad \text{. . . . .} \quad (4.)$$

38. iv. When light passes from one part of a medium whose density is ϱ, to another part of the same medium whose density is ϱ',

$$\text{sine incidence} : \text{sine refraction} = \sqrt{1 + k\varrho'} : \sqrt{1 + k\varrho},$$

k being the *refractive power* of the medium. This optical principle is derived from experience.

Let $\varrho' = \varrho + \delta\varrho$, then

$$\frac{\text{sin incidence}}{\text{sin refraction}} = \frac{\sqrt{1 + k\varrho + k\delta\varrho}}{\sqrt{1 + k\varrho}}.$$

* I specify this, because at first sight it would seem as if the flexure in question taking place round a normal, which is itself continually changing, would not express the due deviation.

39. v. Now let the ray of light be conceived to return in fig. 4. by the path O A A′, or in fig. 5. through $d\,c\,b$, then the angle of incidence is v' (fig. 4.) and the angle of refraction is (by fig. 5.) $v' + \psi$, or by eq. (4.) $v' + \delta\,\theta$. Hence

$$\frac{\sin \text{incidence}}{\sin \text{refraction}} = \frac{\sin v'}{\sin (v' + \delta\,\theta)} = \frac{\sqrt{1 + k\,\varrho + k\,\delta\,\varrho}}{\sqrt{1 + k\,\varrho}} \quad . \quad . \quad . \quad . \quad (5.)$$

$$\sin v' : \sin (v' + \delta\,\theta) = \sqrt{1 + k\,\varrho + k\,\delta\,\varrho} : \sqrt{1 + k\,\varrho}.$$

And passing to differentials,

$$\sin v' : \cos v'\,d\,\theta = \sqrt{1 + k\,\varrho} : -\tfrac{1}{2}\,\frac{k\,d\,\varrho}{\sqrt{1 + k\,\varrho}},$$

$$\frac{\cos v'}{\sin v'}\,d\,\theta = -\,\frac{k\,d\,\varrho}{2\,(1 + k\,\varrho)} \quad . \quad . \quad . \quad . \quad (6.)$$

$$d\,\theta = -\,\frac{k\,d\,\varrho}{2\,(1 + k\,\varrho)}\,\tan v' \quad . \quad . \quad . \quad . \quad (7.)$$

40. vi. Let us now compare equations (1.) and (7.), being the differential values of extinction and of refraction. We observe first, that on the hypothesis of a uniform temperature throughout the atmosphere, the logarithmic law which connects density with height gives us the relation

$$-\,d\,\varrho : \varrho = d\,r : l,$$

the logarithmic subtangent or height of the equiponderant column. Hence

$$\varrho\,d\,r = -\,l\,d\,\varrho \quad . \quad . \quad . \quad . \quad (8.)$$

Substituting, equation (1.) becomes

$$\frac{d\,\varepsilon}{\varepsilon} = d\,.\log \varepsilon = \mathrm{Q}\,l\,d\,\varrho\,\sec v' \quad . \quad . \quad . \quad . \quad (9.)$$

Eq. (7.) gives for the element of refraction

$$d\,\theta = -\,\frac{k\,d\,\varrho}{2\,(1 + k\,\varrho)}\,\tan v'\,;$$

dividing this by the former,

$$\frac{d\,\theta}{d\,.\log \varepsilon} = -\,\frac{k}{2\,\mathrm{Q}\,l\,(1 + k\,\varrho)}\,\sin v' \quad . \quad . \quad . \quad . \quad (10.)$$

41. vii. It remains to determine the value of $\sin v'$.

By eq. (3.) $\qquad d\,v' = d\,\theta - d\,v \quad . \quad . \quad . \quad . \quad (11.)$

By fig. 4.

$$\left.\begin{aligned} \mathrm{A}'\,\mathrm{F} &= r\,d\,v \\ \mathrm{A}'\,\mathrm{F} &= d\,r\,.\tan v' \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (12.)$$

Equating the two last,

$$d\,v = \frac{d\,r}{r}\,\tan v' \quad . \quad . \quad . \quad . \quad (13.)$$

Substituting in eq. (11.) from (7.) and (13.)

$$\frac{d\,v'}{\tan v'} = -\,\left\{\frac{k\,d\,\varrho}{2\,(1 + k\,\varrho)} + \frac{d\,r}{r}\right\},$$

$$\frac{-\cos v'\,d\,v'}{\sin v'} = \frac{d\,r}{r} + \frac{k\,d\,\varrho}{2\,(1 + k\,\varrho)}.$$

Integrating,

$$\log \mathrm{C} - \log \sin v' = \log r + \log \sqrt{1 + k\varrho},$$

C being a constant,

$$\frac{\mathrm{C}}{\sin v'} = r\sqrt{1 + k\varrho} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14.)$$

When the ray starts from O (fig. 4.) the angle at which it intersects the inferior stratum is the apparent zenith distance $= \Theta$; therefore $v' = \Theta$; let also $r = a$ the earth's radius, and $\varrho = (\varrho)$ the density at the earth's surface. Under these circumstances

$$\frac{\mathrm{C}}{\sin \Theta} = a\sqrt{1 + k(\varrho)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15.)$$

Dividing by (14.)

$$\frac{\sin v'}{\sin \Theta} = \frac{a}{r} \frac{\sqrt{1 + k(\varrho)}}{\sqrt{1 + k\varrho}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16.)$$

42. viii. Substituting this value of $\sin v'$ in eq. (10.) it becomes

$$\frac{d\,\theta}{d \,.\log \varepsilon} = \frac{-k}{2\,\mathrm{Q}\,l} \cdot \frac{a}{r} \sqrt{\frac{1 + k(\varrho)}{(1 + k\varrho)^3}} \,.\, \sin \Theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17.)$$

In which, if we consider $k\ (\varrho)$ and $k\,\varrho$ each as small compared to unity, and $\frac{a}{r}$ as nearly equal to unity, the coefficient of $\sin \Theta$ on the second side will be constant for any value of the zenith distance. Also these approximations will be as exact near the horizon as elsewhere. Hence eq. (17.) may be written*

$$d \,.\log \varepsilon = -\frac{\mathrm{H} \,.\, d\theta}{\sin \Theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18.)$$

Integrating and supplying the constant E, and calling now $\delta\,\theta$ the *whole* refraction,

$$\log \frac{\mathrm{E}}{\varepsilon} = \frac{\mathrm{H}\,\delta\,\theta}{\sin \Theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19.)$$

Here E is the value of ε when refraction commences at the exterior limit of the atmosphere; it is therefore the measure of the intensity of solar light there†.

43. ix. From the expression (19.) we may deduce the intensity proper to any altitude from two observations, as by BOUGUER's method.

Let Θ_1 Θ_2 be the apparent zenith distances;
$\delta\,\theta_1$ $\delta\,\theta_2$ the corresponding refractions;
ε_1 ε_2 the corresponding intensities of transmitted light observed.

By eq. (19.)

$$\log \frac{\mathrm{E}}{\varepsilon_1} - \log \frac{\mathrm{E}}{\varepsilon_2} = \mathrm{H} \left\{ \frac{\delta\,\theta_1}{\sin \Theta_1} - \frac{\delta\,\theta_2}{\sin \Theta_2} \right\},$$

$$\log \frac{\mathrm{E}}{\varepsilon_1} - \log \frac{\mathrm{E}}{\varepsilon_2} = \log \frac{\varepsilon_2}{\varepsilon_1} \text{ is known from observation. Let it } = \mathrm{R}:$$

* Méc. Cél., IV. 283.

† E and ε have the same meanings here as V and v in a former part of this paper. In the Mécanique Céleste E is supposed $= 1$, and that letter is used to denote what we have used $[\varepsilon]$ for, farther on.

hence

$$H = R \left\{ \frac{\delta\,\theta_1}{\sin\Theta_1} - \frac{\delta\,\theta_2}{\sin\Theta_2} \right\}^{-1} \quad \ldots \ldots \quad (20.)$$

Also by eq. (19.)

$$\log E = \log \varepsilon_1 + H \frac{\delta\,\theta_1}{\sin\Theta_1} \quad \ldots \ldots \quad (21.)$$

whence the intensity without the atmosphere becomes known.

For any other zenith distance Θ_n,

$$\log \varepsilon_n = \log E - H \frac{\delta\,\theta_n}{\sin\Theta_n} \quad \ldots \ldots \quad (22.)$$

44. x. This expression fails at the zenith; but for all considerable elevations above the horizon the thicknesses of air traversed are as the secants of the zenith distances, and consequently the logarithm of the loss of light is in the same proportion. (See eq. (1.) p. 227). If $[\varepsilon]$ represent the intensity after a vertical transmission, and ε_1 that at an elevation of 45°,

$$\log \frac{E}{[\varepsilon]} : \log \frac{E}{\varepsilon_1} = 1 : \sec 45°,$$

$$= 1 : \sqrt{2},$$

$$\log \frac{E}{[\varepsilon]} = \sqrt{\frac{1}{2}} \cdot \log \frac{E}{\varepsilon_1} \quad \ldots \ldots \quad (23.)$$

For instance, BOUGUER's result for a vertical transmission, $\frac{E}{[\varepsilon]}$ is ·8123. Substituting this in eq. (23.) the intensity at alt. 45° would be found; and for any other elevation the logarithm of the intensity compared to E would be found to be proportional to the refraction divided by the sine of the apparent zenith distance. From what has just been said, the atmospheric masses intercepted are in the same proportion as the logarithms of the intensities. Thus, let μ_1 and μ_2 be the masses of air traversed at 45° of elevation and at any other angle:

$$\mu_1 : \mu_2 = \frac{\delta\,\theta_1}{\sin 45°} : \frac{\delta\theta_2}{\sin\Theta_2}.$$

Taking the value of the refraction from IVORY's latest Table*,

$$\mu_1 : \mu_2 = \frac{58''{\cdot}36}{\sqrt{\frac{1}{2}}} : \frac{\delta\,\theta_2}{\sin\Theta_2}.$$

For example, at the horizon, where $\delta\,\theta = 2072''$, the thickness at the zenith being unity, and therefore that at 45° = $\sqrt{2}$,

$$\mu_2 = \frac{2072''}{58''{\cdot}36} = 35{\cdot}5034.$$

45. The other thicknesses are computed (always with reference to the perpendicular mass of the atmosphere) in the following Table: 1. by the approximate law of the secants; 2. by LAPLACE's analogy; 3. from BOUGUER's Table contained in his 'Optics,' which it will be seen scarcely differs from LAPLACE's result at the horizon. The first

* Philosophical Transactions, 1838.

column contains the zenith distance; the second, its secant; the third, the refraction by IVORY's Table; the fourth, the thickness by the formula $\frac{\delta\theta}{58''\cdot36 \sin\Theta}$; the fifth, the corresponding mass of air in millimetres of mercury; the sixth, the thickness by BOUGUER's Table.

Table of Atmospheric Thicknesses corresponding to different Elevations.

$$\left[B = \frac{\text{Refraction}}{58''\cdot36 \times \sin \text{Z. D.}}\right]$$

Z. D.	Sec. Z. D.	Refraction.	B.	B × 760^{mm}.	BOUGUER's formula.
° ′ 0	1·0000	″ 0·00	1·0000	760·0	1·0000
10	1·0154	10·30	1·0164	772·4	1·0153
20	1·0642	21·26	1·0651	809·5	1·0642
30	1·1547	33·72	1·1556	878·2	1·1547
40	1·3054	48·99	1·3060	992·5	1·3050
50	1·5557	69·52	1·5550	1181·7	1·5561
60	2·0000	100·85	1·9954	1516·4	1·9903
70	2·9238	159·16	2·9023	2205·8	2·8998
75	3·8637	214·70	3·8087	2894·7	3·8046
77 30	4·6202	257·74	4·5237	3438·0	
80	5·7588	320·19	5·5711	4234·0	5·5600
82 30	7·6613	418·59	7·2343	5498·1	
85	11·4737	593·96	10·2165	7762·7	10·2002
86	14·3356	707·43	12·1512	9234·9	12·1401
87	19·1073	866·76	14·8723	11303·0	14·8765
88	28·6537	1101·35	18·8825	14350·7	19·0307
89	57·2987	1466·8	25·1374	19104·4	25·8067
90	infinite	2072·	35·5034	26982·4	35·4955

SECTION IV.—*Account of the following Observations.*

46. In 1832 Sir JOHN HERSCHEL was good enough to direct my attention to the important use which might be made of his actinometers, to find the loss of solar radiation by simultaneous observations at the top and bottom of a mountain; and furnished with two instruments marked G. 7 and B. 2, which had formerly been used by Captain FOSTER in the arctic regions, I made very numerous observations in Switzerland and elsewhere that summer*.

47. We have already seen that, upon certain postulates (such as the uniform opacity of the air), the diminution of the effect of solar radiation in passing through the atmosphere may be ascertained by observations at any station, at varying altitudes. But it is certainly very interesting to test and confirm this indirect result by *simultaneous* observations at different heights in the atmosphere. By this means too, the influence of the meteorological conditions of a column of air directly under experimentmay be immediately ascertained. Balloon observations would theoretically be the most satisfactory, but they appear to offer practical difficulties, in this case nearly insuperable. Simultaneous observations at the top and bottom of a high insulated mountain were

* See Note A. at the end of this paper on the Scale of the Actinometers.

therefore indicated by Sir JOHN HERSCHEL as the most proper for "ascertaining the *very important point* of the comparative force of solar radiation at great and small elevations in the atmosphere*."

48. The difficulties in the way of such an attempt are greater than would at first sight appear, and probably will ever render *satisfactory* observations of this kind very rare. Two practised and zealous observers must agree to devote a considerable time to the experiment; for the assurance of fine weather to a degree that is very seldom met with in mountainous regions, is the first essential. The selection of a station elevated and insulated, and affording a *permanent* shelter to await the opportunity of making the observation, and of making experiments continuously when it arrives, is of the highest importance. At great heights on insulated mountains such stations are exceedingly rare indeed. The instruments employed must be rigorously compared, and the indications afterwards very carefully reduced. All these essentials were in a good measure united in the summer of 1832, and it will appear that I have not exaggerated the difficulties when I state that, among observations made at intervals for some weeks, those of one day only seem sufficiently perfect to yield consistent and trustworthy results†. Considering the value of that day's observations, I have spared no pains in analyzing them as completely as possible, both for the sake of the conclusions they afford, and also to point out for the encouragement of future observers, how well really good observations repay the labour of a detailed reduction. In meteorology, the making of observations is usually by far the least considerable part of the philosopher's task.

49. I was fortunate enough, not only to be provided with instruments and full instructions by Sir JOHN HERSCHEL, but likewise to make the acquaintance of a most zealous and able coadjutor, Professor KÄMTZ of Halle, who was about to proceed from Geneva (where we accidentally met) to an elevated insulated summit in the Oberland of Berne, called the Faulhorn, for the purpose of prosecuting meteorological observations for some weeks. I explained my objects of investigation, and he generously offered his best assistance, and soon acquired a knowledge of the instrument and its use. The month of September was the one he had selected. The Faulhorn is a hill or mountain, which lies exactly between the valley of Grindelwald and the Lake of Brientz. It is perfectly insulated, and commanding fine views in all directions, it has been found worth while to erect a small inn upon the summit, inhabited during a short part of the summer, where travellers can be accommodated. Its height above the sea is 8747 English feet‡. The barometer stands at $21\frac{1}{2}$ English inches: consequently nearly one-third of the atmosphere was left below. The comparative observations were chiefly made at Brientz, on the lake of that name, which has an elevation of only 1903 English feet§, consequently 6844 feet below the Faulhorn: and the difference of barometers (which had been compared and found accurately to

* Private letter of 5th August 1832.

† See Note B. at the end of this paper.

‡ HOFFMAN in BRUGIERE's Orographie; = 2666 metres.

§ Tralles.

agree) was six inches and seven lines French, or above seven English inches, being nearly one-fourth of the whole weight of the atmosphere. It was to be expected then, that if the opacity of the atmosphere for the heating rays at all approached the estimate of LAMBERT, the difference would be very sensible indeed, especially when in consequence of the rays passing through the interposed stratum of 6800 feet with a considerable obliquity, the resistance to their passage was magnified.

50. The month of September was rather changeable, and a fall of snow occurred about the middle of it, which affected so unfavourably the state of the atmosphere, that, though apparently clear weather followed, the results deducible from the observations were of the most anomalous kind. All the observations which the weather enabled us simultaneously to make at the top of the Faulhorn and at Brientz, Grindelwald, or other places in the neighbouring valleys, have been carefully and exactly reduced and computed; but it was not till near the close of the month that the atmosphere appeared to settle into a pure and steady autumnal condition. The 24th, 25th and 26th were brilliant and, so far as I observed, perfectly cloudless days. The 24th I spent partly with M. KÄMTZ on the Faulhorn. In the course of the 25th the observations were continued from the morning till sunset by M. KÄMTZ on the Faulhorn, and by myself at Brientz. To this series of comparative results the chief attention will be drawn. Those on the Faulhorn were made entirely in the open air, those at Brientz, until one o'clock inclusive, were made in a room, and afterwards in the open air, a discrepancy not altogether favourable to the series. At every hour the state of the atmosphere was ascertained by the barometer, thermometer, and moistened bulb hygrometer at both stations, thus giving as accurate a knowledge as the circumstances permitted, of the state of the intercepted column.

51. It is evident that these observations, immediately to be detailed, may be treated in *three* ways entirely distinct. The opacity of the atmosphere may be deduced from observations at different hours at the *upper* station only, by BOUGUER's formula founded on the logarithmic law of the decrement of solar intensity; the observations at the *lower* station give independent data; and finally, the direct determination of the loss of solar heat in passing from the upper to the lower station, by a comparison of the two instruments, yields a distinct result exclusively derived from the action of the lower strata of the atmosphere in absorbing solar heat. For this last purpose then a rigorous comparison of the arbitrary scale of the two instruments is of the highest importance. The following ample series of experiments will show the degree of accuracy attainable in such observations.

Comparison of Actinometers.

52. The comparison of the arbitrary scales of the two actinometers marked B. 2. and G. 7, was obtained by making alternating sets of observations on the solar intensity with each. It has already been stated that the method of using the instrument is to expose it to the sun for a certain short time (say one minute), then to interpose a

screen between the sun and the actinometer, and observe the effect of all other influences besides solar radiation for an equal space of time; and so on alternately subtracting algebraically the shade-effect from the sun-effect, so as to get the total influence of the solar rays. An example will make this clearer.

Paris.—M. ARAGO's Magnetic Cabinet, June 10, 1833.

Actinometer.	Hour.	Sun or shade.	Reading.	Diff.	Mean shade effect.	Sun-effect *minus* shade-effect.
B. 2.	h m s 12 26 0 27 0	shade.	18·3 10·9	− 7·4		
	27 0 28 0	⊙	10·9 40·2	29·3	−6·7	36·0
	28 0 29 0	shade.	40·2 34·2	− 6·0		
	29 0 30 0	⊙	34·2 64·3	30·1	−6·3	36·4
	30 0 31 0	shade.	64·3 57·7	− 6·6		
G. 7.	32 0 33 0	shade.	11·6 4·4	− 7·2		
	33 0 34 0	⊙	4·4 28·8	24·4	−6·85	31·25
	34 0 35 0	shade.	28·8 22·3	− 6·5		
	35 0 36 0	⊙	22·3 46·6	24·3	−6·45	30·75
	36 0 37 0	shade.	46·6 40·2	− 6·4		
B. 2.	38 0 39 0	shade.	33·7 23·5	−10·2		
	39 0 40 0	⊙	23·5 50·7	27·2	−9·3	36·5
	40 0 41 0	shade.	50·7 42·3	− 8·4		
	41 0 42 0	⊙	42·3 70·7	28·4	−8·55	36·95
	42 0 43 15	shade.	70·7 59·8	− 8·7*		

53. The following are the results of various series of observations similarly conducted.

* Reduced to 1^{m} interval.

I. Geneva, August 18, 1832.

			Ratio. B. 2. : G. 7.
Actinometer B. 2.	26·3		
G. 7.	22·4	26·3	1·174
B. 2.	26·3		
G. 7.	23·8	26·55	1·116
B. 2.	26·8		
G. 2.	23·9	26·8	1·121
		Mean	1·137

II. Geneva, September 5, 1832.

		Mean.	
B. 2.	18·0		
G. 7.	16·5	19·8	1·200
B. 2.	21·6		
G. 7.	19·9	21·9	1·100
B. 2.	22·2		
G. 7.	23·2	23·2	1·000
B. 2.	24·8		
G. 7.	20·0	22·5	1·125
B. 2.	20·8		
G. 7.	21·9	22·2	1·014
B. 2.	23·7		
		Mean	1·088

The differences here are evidently due to variations in the state of the atmosphere.

III. Faulhorn, September 24, 1832 (by M. Kämtz).

B. 2.	23·0 ; 22·7 ; 24·1	Mean 23·26		
G. 7.	17·2 ; 17·1 ; 17·7	Mean 17·33	24·14	1·393
B. 2.	23·8 ; 25·7 ; 25·6	Mean 25·03		
G. 7.	20·9 ; 21·3 ; 21·1 ; 22·2	Mean 21·37	26·3	1·231
B. 2.	27·3 ; 27·4 ; 27·8 ; 27·8	Mean 27·57		
			Mean	1·312

IV. Faulhorn, September 24 (Kämtz).

G. 7.	21·7 ; 24·2 ; 24·9 ; 24·9	Mean 23·92		
B. 2.	27·2 ; 28·8 ; 29·1 ; 29·1	Mean 28·55	25·04	1·140
G. 7.	25·4 ; 26·1 ; 27·0	Mean 26·16		

V. Paris, June 10, 1833. In M. Arago's Magnetic Cabinet.

G. 7.	29·9 ; 28·8	Mean 29·35		
B. 2.	35·8 ; 35·2	Mean 35·5	30·02	1·182
G. 7.	30·2 ; 31·2	Mean 30·7		
B. 2.	36·0 ; 36·4	Mean 36·2	30·82	1·174
G. 7.	31·2 ; 30·7	Mean 30·95		
B. 2.	36·5 ; 36·9	Mean 36·7	31·1	1·180
G. 7.	31·3 ; 31·2	Mean 31·25		
B. 2.	36·6 ; 35·1	Mean 35·85	30·92	1·159
G. 7.	30·8 ; 30·6	Mean 30·7		
B. 2.	34·4 ; 35·1	Mean 34·75	30·32	1·146
G. 7.	28·5 ; 31·4	Mean 29·95		
B. 2.	34·8 ; 35·5	Mean 35·15	30·15	1·166
G. 7.	30·6 ; 30·1	Mean 30·35		
			Mean	1·168

54. The mean of the whole, giving to each series its proper weight, is 1·154; but considering the variations of the second, third, and fourth series, and the extremely favourable circumstances and good agreement of the fifth, I prefer to adopt *it* alone, and assume for the factor of reduction G. 7. to B. 2. . . . . 1·168*.

Reduction to Intervals of One Minute.

55. In the instructions which Sir J. HERSCHEL had provided for me, it was observed that the velocity of heating or cooling might be noted for thirty seconds instead of one minute, and reduced to the standard unit by doubling it. But this does not appear to be exact, and in order to compare observations made with thirty-second intervals with those made at sixty-second intervals, a greater factor than 2 appears to be necessary, for reasons not difficult to anticipate. Whilst therefore I agree with the later instructions in preferring sixty-second intervals, it is useful to have a factor for reduction†.

56. Professor KÄMTZ, by careful and multiplied observations on the actinometer B. 2. at the Faulhorn, found the rise in 15, 30, and 60 seconds, to be proportionally 1, 2·345, 5·208 in the MS. notes with which he supplied me, and which almost exactly coincides with what he has stated in his work on Meteorology, vol. iii. p. 21. Hence the factor of reduction of thirty-second to sixty-second intervals is $\frac{5 \cdot 208}{2 \cdot 345} = 2 \cdot 224$, which I have employed when necessary.

57. The following Tables contain the meteorological observations on the 25th of September 1832, at Brientz and the Faulhorn, with the reductions necessary to render the results immediately applicable.

58. The observed times were nearly mean time at the place. They are reduced to *apparent time*, which is used in all the calculations and projections in which the actinometer is employed, on account of the facility which it affords for the direct comparison of observations at equal altitudes before and after noon.

59. The barometers used were both on the syphon construction: that at the lower station was divided on the old Swiss plan of French inches, lines, and 16ths with double readings. The upper barometer was divided into French lines and decimals, and was reduced to zero, REAUMUR, by M. KÄMTZ, before communicating the readings to me. I have reduced the other to the same temperature by means of its attached thermometer. Both barometers have been reduced into millimetres, which has been assumed as the standard of calculation (and 760 millimetres as the mean atmospheric pressure) for reasons which I need not now specify. The barometers were compared on the 24th of September and found to agree. Their index errors, as

* The observations at Geneva (first series), which are the best of the others, show that there is no reason for believing that the instruments had changed in any way at the date of the fourth series.

† Where by accident a single observation has been extended to seventy or seventy-five seconds, a simple proportional reduction will be sufficient, as shown in one of the examples already quoted.

well as those of the thermometers (which are all by good makers), are not recorded, but can hardly have an appreciable influence on any of the results about to be deduced.

60. The detached and moistened thermometers are reduced to FAHRENHEIT'S scale, and the absolute elasticity of vapour in inches of mercury, as well as its hygrometric state relative to absolute saturation, are calculated from Dr. APJOHN'S formula and tables*. The formula is

$$e'' = e' - \frac{1}{87}(t - t')\frac{b}{30},$$

where t and t' are the readings (FAHRENHEIT) of the dry and wetted thermometer; e' the maximum elasticity of vapours corresponding to t'; e'' that corresponding to the dew-point; b the observed height of the barometer in English inches†. The hygrometric observations have considerable interest in themselves owing to the extraordinary dryness of the air at the upper station,—a dryness, it is believed, altogether unusual even at that elevation; being an elasticity of vapour at $7\frac{1}{2}^{h}$ A.M. of only ·038 inch at temperature 39°, or ratio to saturation of ·148. This dryness must be considered as one of the peculiarly favourable circumstances of the present experiment.

TABLE A.—Meteorological Observations at Brientz, September 25, 1832.

Mean time.	Apparent time.	Barometer, French.		Attached Therm. REAUM.	Barometer in millimetres.	Attached Therm. Cent.	Barometer at 0° C. mm.	Detached Therm. FAHR.	Moist Therm. FAHR.	Diff.	Elasticity of vapour in inches of mercury.	Relative dampness.
h m	h m	inches.	lines 16ths.	°				°	°	°		
8 2	8 10	26	10·8	14·6	727·51	18·25	725·06	55·0	51·8	3·2	·362	$\frac{362}{442}$ = ·819
9 5	9 13	26	10·8	14·7	727·51	18·38	725·07	58·1	54·7	3·4	·401	$\frac{401}{491}$ = ·817
10 5	10 13	26	10·8	15·0	727·51	18·75	724·99	60·5	55·5	5·0	·395	$\frac{395}{532}$ = ·743
11 3	11 11	26	10·4	15·3	726·94	19·13	724·38	61·2	56·5	4·7	·414	$\frac{414}{545}$ = ·760
11 50	11 58	26	9·13	15·0	725·94	18·75	723·43	65·5	57·2	8·3	·386	$\frac{386}{628}$ = ·615
1 1	1 9	26	9·14	15·0	726·10	18·75	723·59	68·0	59·0	9·0	·408	$\frac{408}{681}$ = ·599
2 17	2 25	26	9·12	15·5	725·81	19·38	723·21	68·4	59·5	8·9	·418	$\frac{418}{690}$ = ·606
3 2	3 10	26	9·11	15·3	725·67	19·13	723·11	64·3	59·5	4·8	·460	$\frac{460}{603}$ = ·763
4 4	4 12	26	9·9	14·9	725·38	18·63	722·89	64·2	57·5	6·7	·408	$\frac{408}{601}$ = ·679
4 38	4 46	26	9·8	14·9	725·25	18·63	722·76	62·2	58·8	3·4	·466	$\frac{466}{563}$ = ·828

* In order to obtain tolerably consecutive results, it was found necessary to project graphically both the dry and moist thermometer observations, Nos. I. II. III. IV. Plate XVIII., and to run curves freely amongst the points. The values for the whole hours are thus obtained in Tables A. and B, and the elasticities of vapour for those hours are thence computed.

† Supplementary Report on Meteorology, British Association Report, 1840, p. 98; and Royal Society's Instructions.

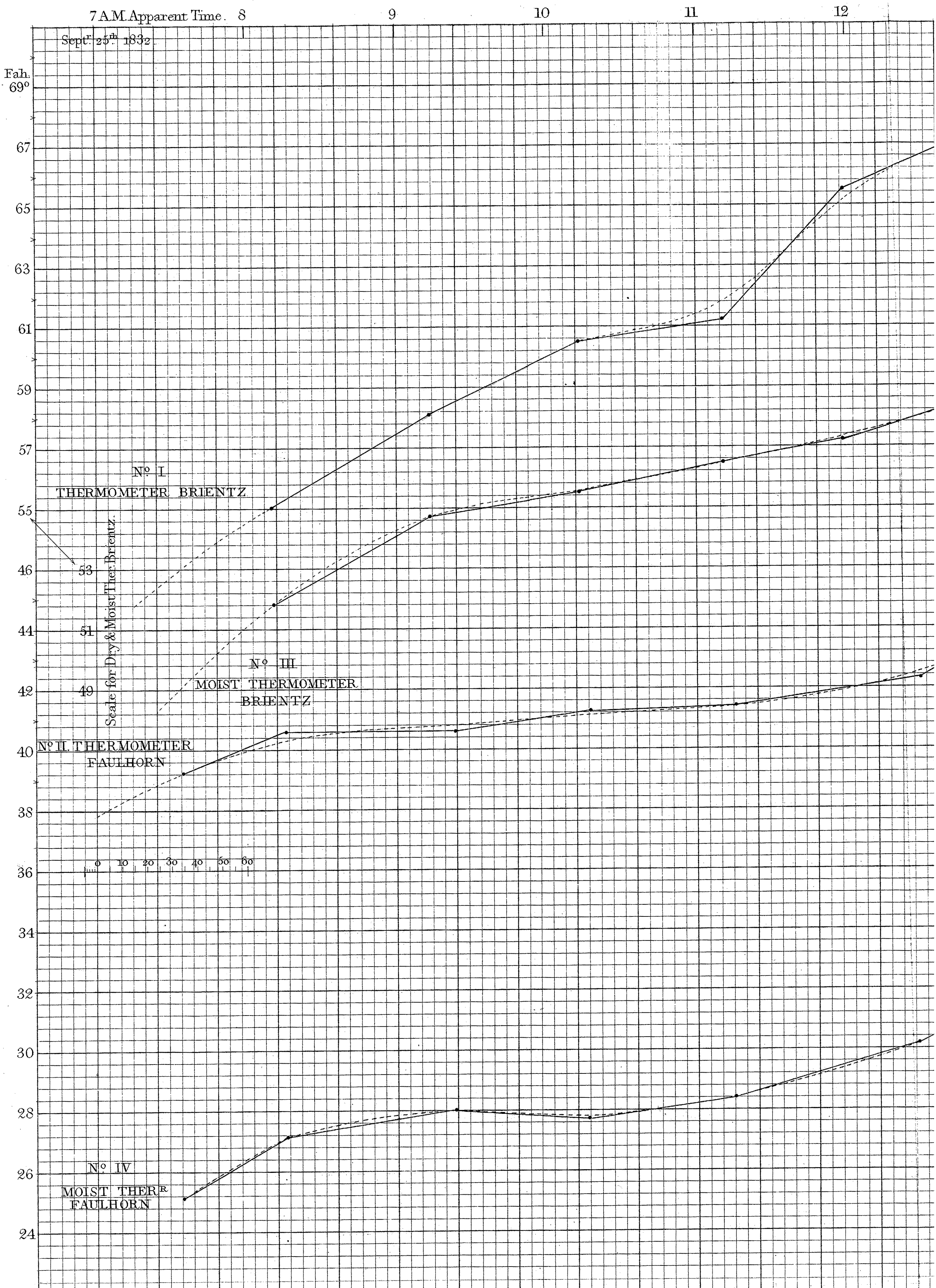
TABLE B.—Meteorological Observations at the Faulhorn, September 25, 1832.

Mean time.	Apparent time.	Barometer, French lines at 0° Cent.	Barometer, millimetres at 0° Cent.	Detached Therm. REAUM.	Moist Therm. REAUM.	Detached Therm. FAHR.	Moist Therm. FAHR.	Diff.	Elasticity of vapour.	Relative dampness.
h m	h m			°	°	°	°	°	inch.	
7 26	7 34	247·13	557·49	3·2	−3·1	39·2	25·1	14·1	·038	$\frac{·038}{·257} =$ ·148
8 7	8 15	247·15	557·53	3·8	−2·2	40·6	27·1	13·5	·053	$\frac{·053}{·270} =$ ·196
9 15	9 23	247·24	557·73	3·8	−1·8	40·6	28·0	12·6	·067	$\frac{·067}{·270} =$ ·248
10 9	10 17	247·34	557·96	4·1	−1·9	41·2	27·7	13·5	·057	$\frac{·057}{·276} =$ ·206
11 7	11 15	247·33	557·94	4·2	−1·6	41·4	28·4	13·0	·065	$\frac{·065}{·278} =$ ·234
0 21	0 29	247·28	557·82	4·6	−0·8	42·3	30·2	12·1	·086	$\frac{·086}{·286} =$ ·301
1 4	1 12	247·18	557·60	5·5	0·0	44·4	32·0	12·4	·096	$\frac{·096}{·308} =$ ·312
2 27	2 35	247·15	557·53	6·4	+2·6	46·4	37·8	8·6	·172	$\frac{·172}{·330} =$ ·521
3 30	3 38	246·99	557·17	5·6	+1·8	44·6	36·1	8·5	·160	$\frac{·160}{·310} =$ ·516
4 16	4 24	246·90	556·99	4·5	+1·5	42·1	35·4	6·7	·169	$\frac{·169}{·284} =$ ·595

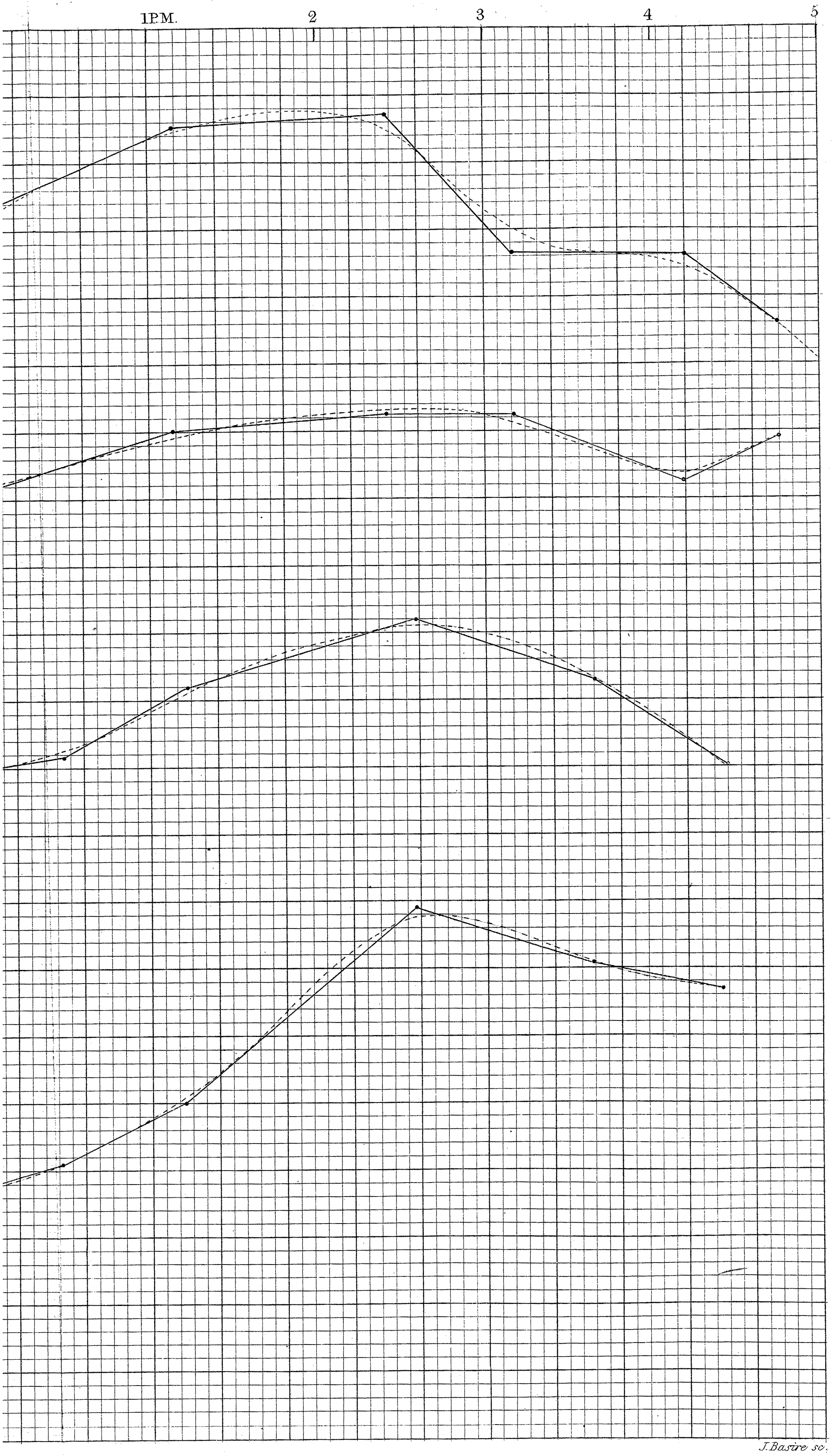
61. The following Tables contain the observations with the actinometers at each station; those at the lower, made with the instrument G. 7, are made comparable to those at the upper with the actinometer B. 2, by the application of the constant factor 1·168 already found. It has been already stated that the day was cloudless; that the observations were made entirely in the open air at the Faulhorn: those at Brientz were made in a room until one o'clock, and afterwards in the open air. For the sake of brevity, the results only are given, and not the readings upon which they are founded.

TABLE C.—Actinometer Observations at Brientz, September 25, 1832.

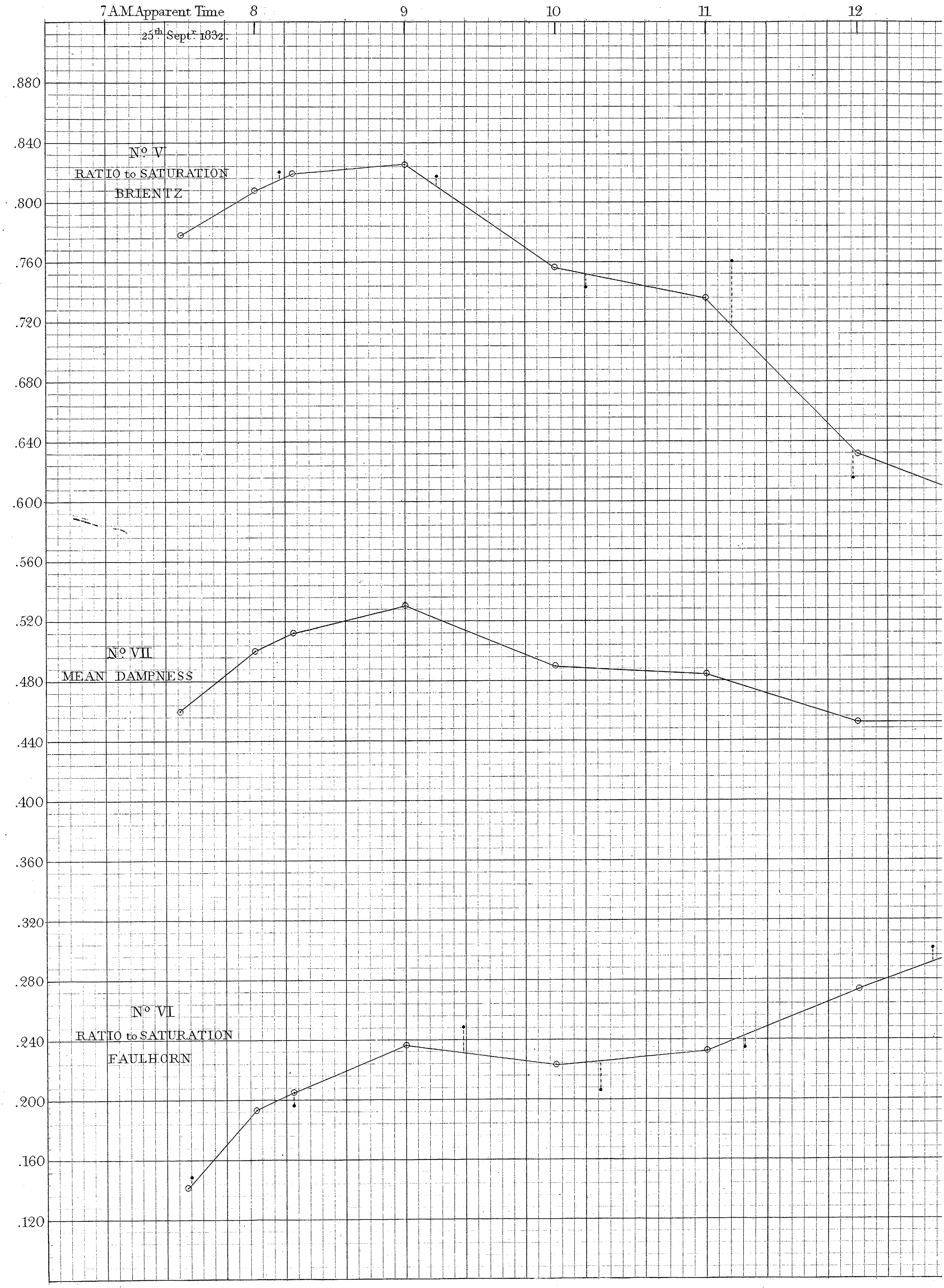
Hour. From	Hour. To	Hour. Mean.	Apparent time.	Intervals observed.	Actinometer G. 7.	Mean.	Reduced to 60 sec.	Reduced to B. 2.	Remarks.
h m	h m	h m	h m	s					
8 2	8 11½	8 6¾	8 14¾	60	18·7; 19·0; 19·7; 20·7;	19·5	19·5	22·8	In a room.
9 5	9 14½	9 9¾	9 17¾	60	22·9; 22·2; 22·9; 22·8;	22·7	22·7	26·5	
10 5	10 15	10 10	10 18	60	23·3; 23·5; 24·4; 24·3;	23·9	23·9	27·9	
11 3	11 15	11 9	11 17	60	24·2; 24·6; 25·2; 25·3; 25·2;	24·9	24·9	29·1	
11 50	12 5½	11 57¾	12 5¾	60	26·0; 26·8; 26·7; 25·8; 26·1; 26·8;	26·4	26·4	30·8	
12 5½	12 19	12 12¼	12 20¼	60	26·3; 25·1; 26·7; 27·1; 27·4; 26·8;	26·9	26·9	31·4	
12 19	12 24½	12 21¾	12 29¾	30	25·4; 25·2; 25·0; 23·3;	24·7	27·5	32·1	
1 1	1 12½	1 6¾	1 14¾	60	26·2;? 26·6; 26·4; 27·7; 27·3;	26·8	26·8	31·3	
2 17½	2 28½	2 23	2 31	60	21·7; 21·3; 21·3; 21·0;	21·3	21·3	24·9	Outofdoors.
3 2	3 11	3 6½	3 14½	60	20·8; 20·7; 20·5; 20·9;	20·7	20·7	24·2	
4 4½	4 15½	4 10	4 18	60	15·8; 15·1; 15·4; 15·3;	15·4	15·4	18·0	
4 38½	4 42½	4 40½	4 48½	60	11·1; 9·9;	10·5	10·5	12·3	

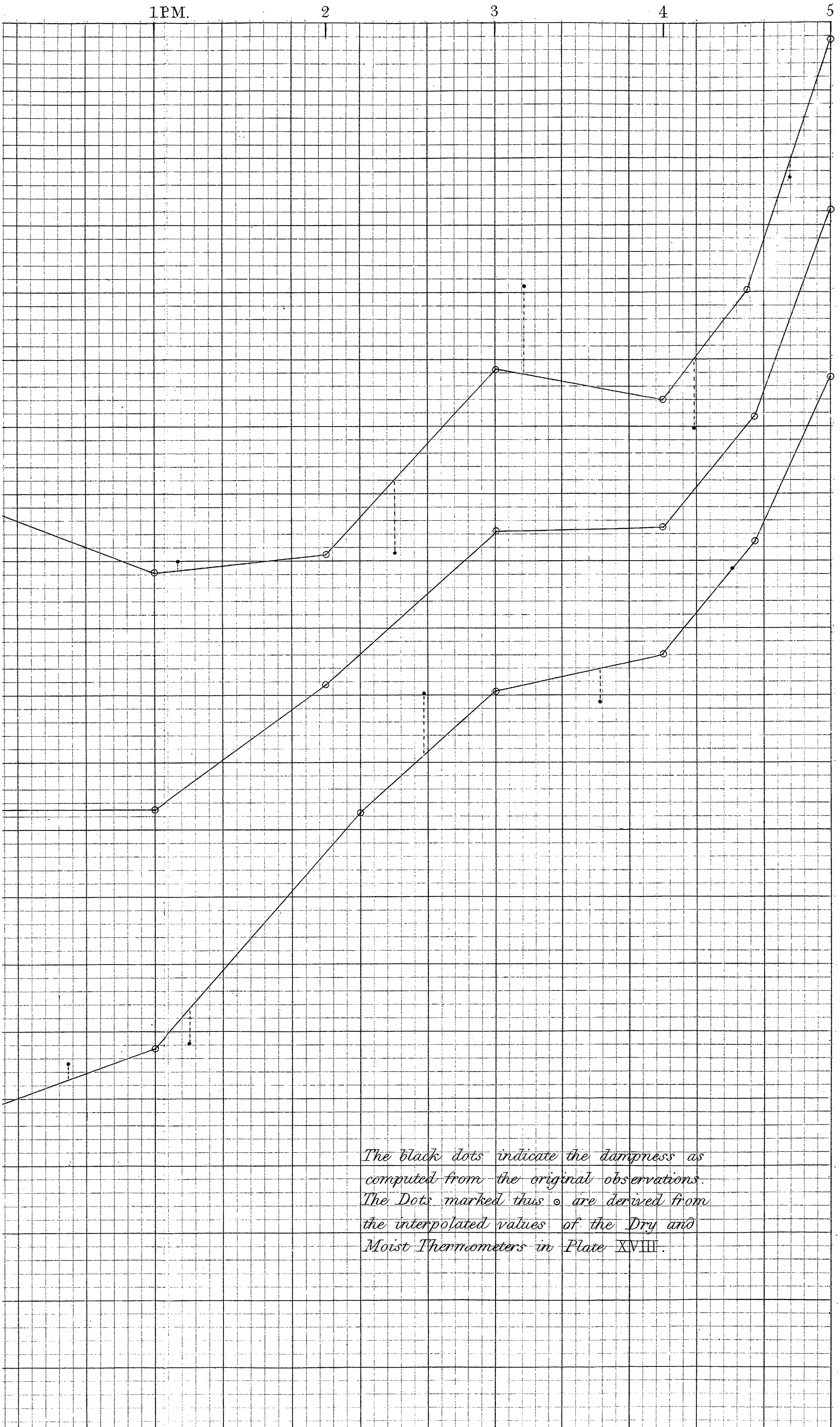


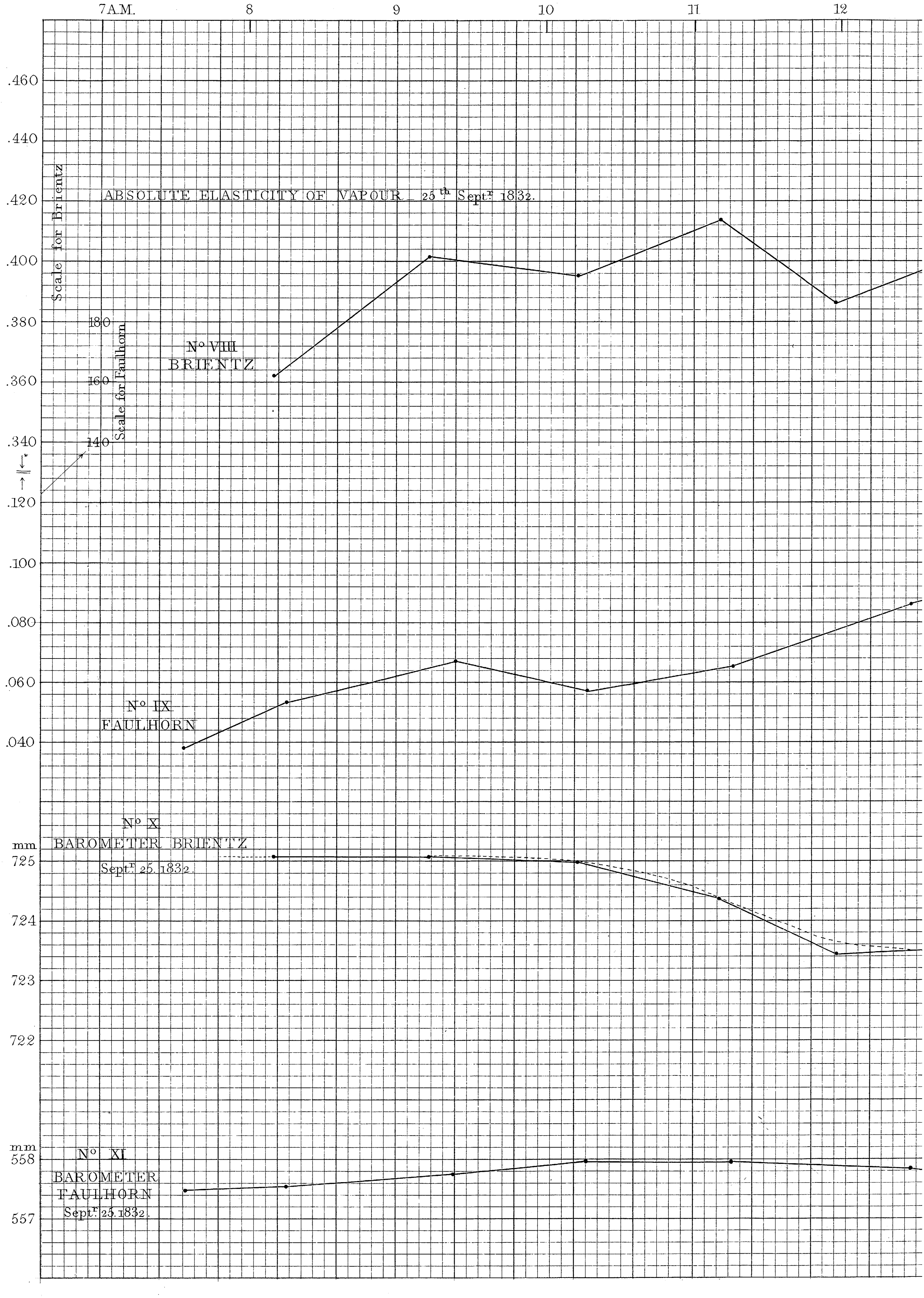
7 A.M. Apparent Time. 8
9
10
11
12
Sept.r 25th 1832.
Fah. 69°
67
65
63
61
59
57
55
46
44
42
40
38
36
34
32
30
28
26
24
53
51
49
Scale for Dry & Moist Ther. Brientz.
No I
THERMOMETER BRIENTZ
No III
MOIST THERMOMETER
BRIENTZ
No II. THERMOMETER
FAULHORN
0 10 20 30 40 50 60
No IV
MOIST THERR
FAULHORN



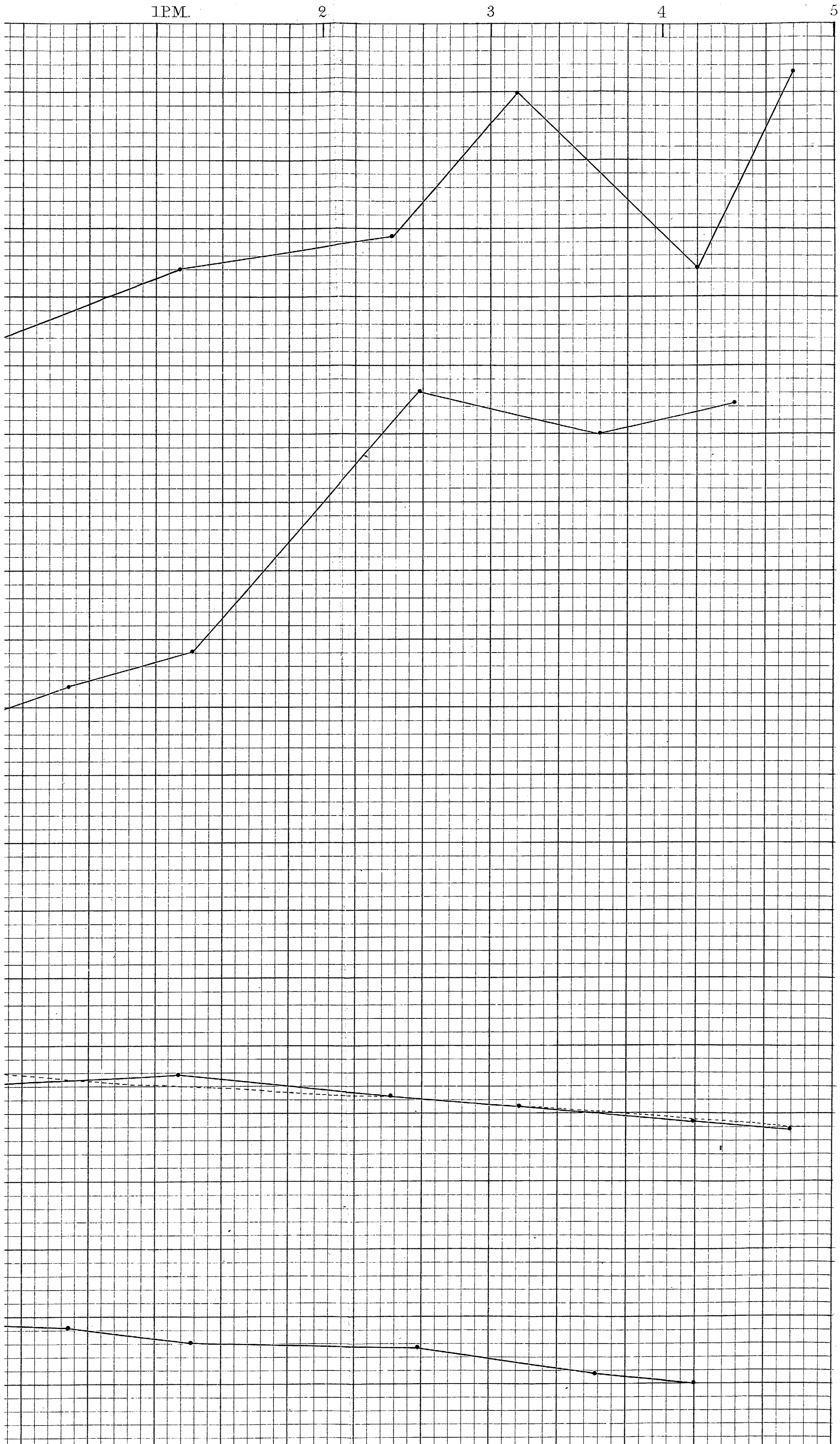
J. Basire sc.

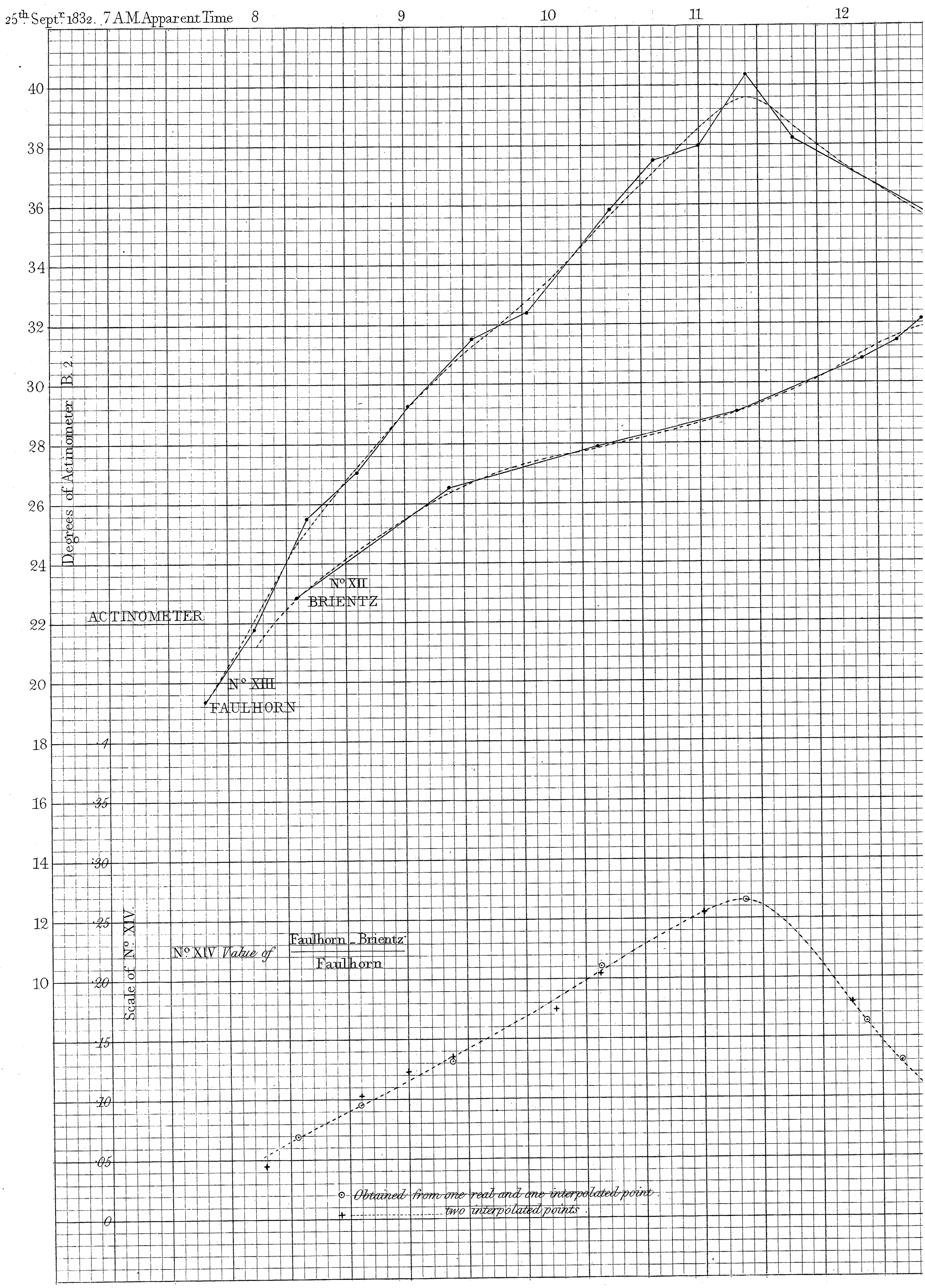

7 A.M. Apparent Time
8
9
10
11
12
25th Septr. 1832.
.880
.840
.800
.760
.720
.680
.640
.600
.560
.520
.480
.440
.400
.360
.320
.280
.240
.200
.160
.120
No. V
RATIO to SATURATION
BRIENTZ
No. VII
MEAN DAMPNESS
No. VI
RATIO to SATURATION
FAULHORN


1 P.M.
2
3
4
5
The black dots indicate the dampness as computed from the original observations.
The Dots marked thus ⊙ are derived from the interpolated values of the Dry and Moist Thermometers in Plate XVIII.
J. Basire sc.

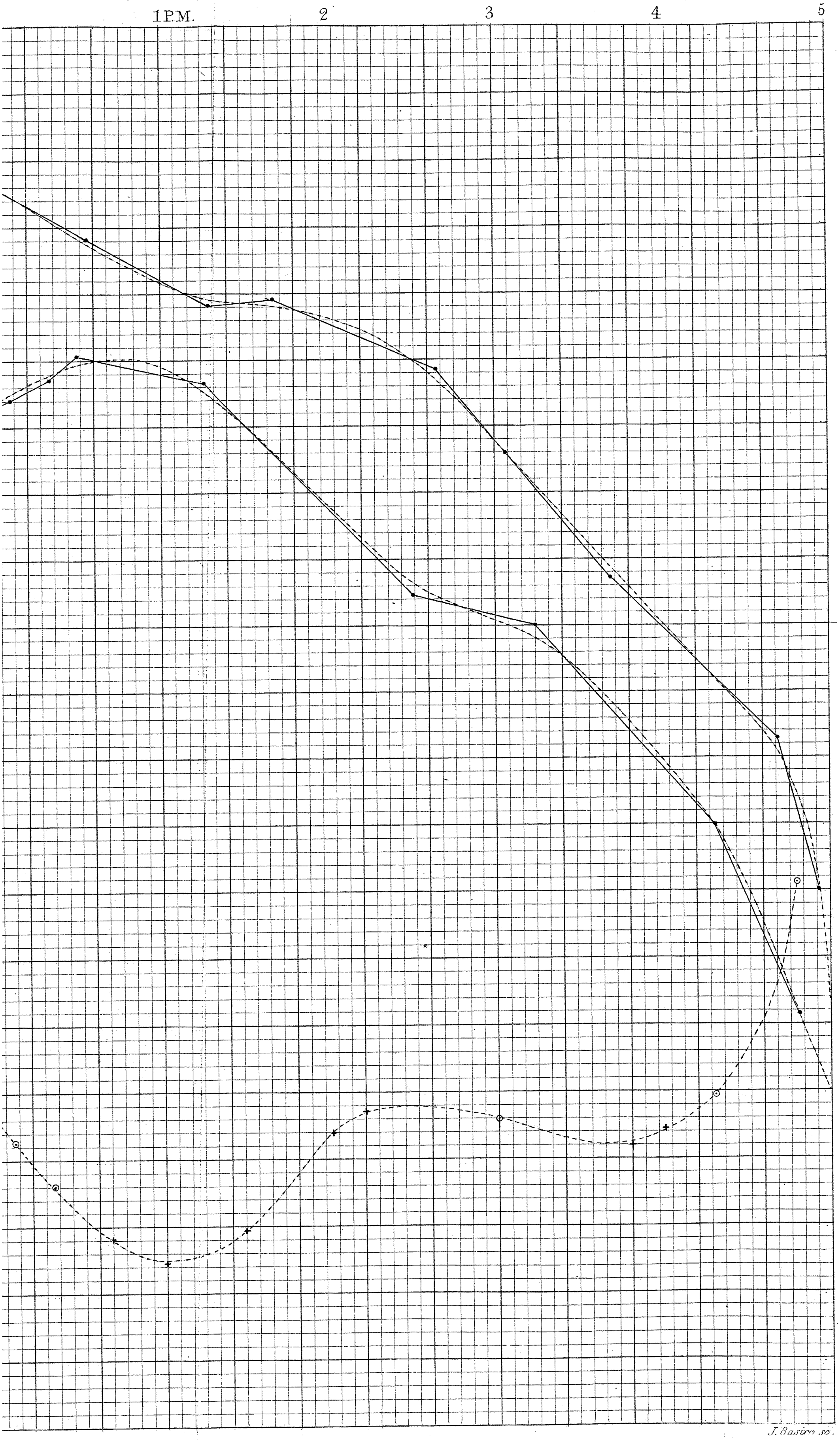


7A.M.
8
9
10
11
12
ABSOLUTE ELASTICITY OF VAPOUR _ 25th Septr. 1832.
Scale for Brientz
Scale for Faulhorn
.460
.440
.420
.400
.380
.360
.340
.120
.100
.080
.060
.040
180
160
140
Nº VIII
BRIENTZ
Nº IX
FAULHORN
Nº X
BAROMETER BRIENTZ
Septr. 25. 1832.
mm
725
724
723
722
Nº XI
BAROMETER
FAULHORN
Septr. 25.1832.
mm
558
557


1 P.M.
2
3
4
5
J. Basire sc.



25th Septr. 1832. 7 A.M. Apparent Time
8
9
10
11
12
40
38
36
34
32
30
28
26
24
22
20
18
16
14
12
10
Degrees of Actinometer B. 2.
ACTINOMETER.
No XII
BRIENTZ
No XIII
FAULHORN
·4
·35
·30
·25
·20
·15
·10
·05
0
Scale of No. XIV.
No XIV Value of Faulhorn _ Brientz / Faulhorn
Obtained from one real and one interpolated point
two interpolated points



J. Basire sc.

TABLE D.—Actinometer Observations at the Faulhorn, September 25, 1832.

Hour. From	Hour. To	Hour. Mean.	Apparent time.	Intervals observed.	Actinometer B. 2.	Mean.	Remarks.
h m	h m	h m	h m	s			
7 26	7 35½	7 30¾	7 38¾	60	16·4; 16·9; 17·6; 18·2;	17·3	
7 46	7 55½	7 50¾	7 58¾	60	21·6; 20·9; 21·7; 22·8;	21·8	
8 7½	8 17	8 12¼	8 20¼	60	25·7; 25·8; 25·4; 25·0;	25·5	
8 28	8 37½	8 32¾	8 40¾	60	26·3; 26·4; 27·9; 27·8;	27·1	
8 48½	8 58	8 53¼	9 1¼	60	27·1; 28·7; 29·2; 31·9;	29·2	
9 15½	9 24½	9 20	9 28	60	30·6; 31·0; 31·8; 32·7;	31·5	
9 37½	9 46½	9 42	9 50	60	31·9; 32·1; 32·5; 33·1;	32·4	
10 9	10 18	10 13½	10 21½	60	36·9; 34·7; 36·0; 35·6;	35·8	
10 29½	10 38¼	10 34	10 42	60	38·0; 36·8; 36·4; 38·6;	37·5	
10 47½	10 56½	10 52	11 0	60	38·7; 37·3; 38·1; 38·0;	38·0	All in the open air.
11 7	11 16¼	11 11½	11 19½	60	39·2; 40·3; 40·8; 41·3;	40·4	
11 26½	11 35½	11 31	11 39	60	38·2; 38·9; 38·2; 37·4:	38·2	
12 21	12 30½	12 25¾	12 33¾	60	35·1; 36·3; 35·3; 35·6;	35·6	
1 4½	1 14	1 9¼	1 17¼	60	32·3; 31·7; 35·1; 35·1;	33·6	
1 27½	1 37	1 32¼	1 40¼	60	32·2; 33·8; 34·2; 34·8;	33·8	
2 27	2 36½	2 31¾	2 39¾	60	31·5; 31·1; 31·0; 33·0;	31·7	
2 47½	2 54½	2 51	2 59	60	29·7; 30·2; 28·9;	29·6	
3 30	3 39	3 34½	3 42½	60	25·0; 26·5; 24·9; 25·2;	25·4	
4 16½	4 25½	4 21	4 29	60	21·3; 20·2; 20·1; 20·6;	20·6	
4 43½	4 52½	4 48	4 56	60	16·5; 16·2; 15·5; 15·8;	16·0	

62. As the observations above and below are nearly contemporaneous, we might readily enough proceed to compare them directly. But I have thought it more exact, and also more instructive, to tabulate the meteorological and other data in the form of curves, and by graphical interpolation to obtain the desired quantities for the *whole* hours, by which means the errors of observation, and also local and momentary atmospheric changes are in some measure eliminated, and a kind of approximation made as to the mean condition, at any moment, of the mass of air under experiment, 6800 feet in thickness, with respect to density, temperature, moisture, and opacity. The great advantages of this method will be seen in the sequel. The curves numbered I. to XIV. in Plates XVIII. to XXI. represent this interpolation, the points of observation being always connected by straight lines, and then a curve drawn easily through them. When reduced again to numbers, we have these regulated data contained in the following Table, from which are deduced,—1st, the intercepted mass of air; 2nd, the mean temperature of the mass of air; 3rd, its mean relative dampness or ratio to saturation; 4th, the loss of solar intensity in the passage of the rays from the level of the upper to the lower station; 5th, the ratio of the intensity at the upper to that at the lower station. This Table also contains,—6th, the sun's *apparent* altitude for every hour of apparent time computed by the usual formula from the hour-angle, corrected for refraction and for the change of declination; 7th, the approximate measure of the total mass of air traversed by the ray with the varying obliquity at each station; 8th, the difference of the two last determinations which gives the effective mass interposed between the stations. These masses of air are supposed to vary as the secant of the zenith distance (see Art. 33.).

TABLE E.—September 25, 1832.

Apparent time.	Brientz.					Faulhorn.					Difference, Dry and Moist Thermometers.		Mean Temp. FAHR.	Difference of Pressure.	[c]	Loss of solar intensity.	Ratio of actinometers $\frac{b}{a}$.	Ratio of loss to effect at Upper Station*.	Sun's apparent altitude.	Barometer × sec. Z. D.		[d]	c × d.
	Barometer at 0° C.	Therm. FAHR.	Moist Therm. FAHR.	Ratio to saturation.	[a] Actinometer, B. 2.	Barometer at 0° C.	Therm. FAHR.	Moist Therm. FAHR.	Ratio to saturation.	[b] Actinometer, B. 2.	Brientz.	Faulhorn.			Mean dampness.					Brientz.	Faulhorn.	Diff.	
	mm					mm								mm					° ′	mm	mm		
7½	725·06	52·8?*	49·0	·779	....	557·49	39·1	24·9	·141	18·2?	3·8	14·2	45·9	167·57	·460	....	......	......	14 42	2857·0	2197·0	660·0	303·6
8	725·06	54·5	51·1	·807	....	557·53	40·0	26·5	·193	22·2	3·4	13·5	47·2	167·53	·500	....	......	......	19 26	2179·0	1675·8	503·2	251·6
8¼	725·06	55·3	52·1	·819	22·8	557·53	40·4	27·1	·205	24·5	3·2	13·3	47·8	167·53	·512	1°7	1·075	·0694	21 48	1952·2	1501·4	450·8	230·8
9	725·06	57·5	54·3	·825	25·5	557·69	40·7	27·9	·236	29·1	3·2	12·8	49·1	167·37	·530	3·6	1·141	·1237	28 18	1529·2	1176·4	352·8	187·0
10	725·03	60·1	55·4	·756	27·6	557·89	41·0	27·8	·223	33·5	4·7	13·2	50·5	167·14	·489	5·9	1·214	·1761	35 39	1244·1	957·3	286·8	140·2
11	724·57	61·5	56·3	·736	28·7	557·96	41·3	28·2	·232	38·6	5·2	13·1	51·4	166·61	·484	9·9	1·345	·2565	40 38	1112·6	856·8	255·8	123·8
12	723·68	65·2	57·3	·631	30·6	557·87	41·9	29·4	·273	37·3	7·9	12·5	53·5	165·81	·452	6·7	1·219	·1796	42 24	1073·2	827·4	245·8	111·1
1	723·42	67·7	58·6	·593	31·8	557·64	43·7	31·5	·310	34·3	9·1	12·2	55·7	165·78	·451	2·5	1·078	·0729	40 37	1111·2	856·7	254·5	114·8
2	723·30	68·5	59·5	·603	27·6	557·55	45·7	35·8	·450	33·3	9·0	9·9	57·1	165·75	·526	5·7	1·207	·1712	35 36	1242·5	957·9	284·6	149·7
3	723·16	65·6	59·5	·713	24·2	557·37	45·9	37·3	·522	29·5	6·1	8·6	55·7	165·79	·617	5·3	1·219	·1796	28 14	1528·7	1178·3	350·4	216·2
4	722·95	64·1	57·8	·696	19·9	557·09	43·5	35·7	·544	24·0	6·3	7·8	53·8	165·86	·620	4·1	1·206	·1708	19 20	2183·7	1683·1	500·6	310·4
4½	722·85	63·1	58·2	·759	16·2	556·91	41·7	35·4	·611	21·4	4·9	6·3	52·4	165·94	·685	5·3	1·321	·2477	14 34	2874·0	2214·5	659·5	451·7
5	722·75?	61·2?	59·5	·910	10·0?	556·73	39·7	35·2	·709	12·0?	1·7	4·5	50·4	166·02	·809	2·0?	1·200?	·1667?	9 34	4348·8	3350·2	998·6	807·9

* Projected in Curve XIV.

SECTION V.—*Analysis of the Observations of the 25th of September*, 1832.

63. Looking first to the *differential* observations, or the comparative results of those at the two stations, we observe with respect to the solar intensities at Brientz and the Faulhorn as contained in Table E, and as projected in Plate XXI.,—1st, that the intensity at the higher station always exceeded that at the lower by a very appreciable quantity, varying from nearly ten to nearly two degrees of the actinometer B. 2; 2nd, that this loss, compared to the intensity at the higher station, varied from $\frac{7}{100}$ or $\frac{1}{14}$th of the total amount, to above $\frac{25}{100}$th or one-fourth of the total amount; 3rd, that this relative loss appears to have varied rather irregularly, having two maxima nearly equal at 11 A.M. and $4\frac{1}{2}$ P.M.

64. If (without inquiring for the moment into the cause and measure of this variation of effect) we simply seek to deduce a mean value for the opacity of the atmosphere for the entire day of the 25th of September, 1832, we may do so in the following manner:—

65. Let the intensity of the sun's rays at the upper station be denoted by 1, and let its varying value be v: then let x measure the mass of air traversed, measured by an equiponderant column in millimetres of mercury; further, let m be a constant. On the hypothesis of uniform opacity,

$$-dv = m \, . \, v \, dx \quad \ldots \ldots \ldots \ldots \quad (1.)$$

$$-\frac{dv}{v} = m \, dx$$

$$\log \frac{1}{v} = m \, x. \quad \ldots \ldots \ldots \ldots \quad (2.)$$

Any number of such observations being made giving corresponding values of v and x, by summation

$$\Sigma \log \frac{1}{v} = m \, \Sigma \, x \, ;$$

whence

$$m = \frac{\Sigma \log \frac{1}{v}}{\Sigma \, x}, \quad \ldots \ldots \ldots \ldots \quad (3.)$$

where m may be a constant adapted to the tabular instead of hyperbolic logarithms.

66. If the measure of opacity sought (which is that to which we shall generally refer) be the residual intensity of the sun's rays, after passing vertically through a mass of air which balances 760 millimetres of mercury, and if this residual intensity be $[v]$, we shall have by (2.)

$$\log \frac{1}{[v]} = \frac{\Sigma \log \frac{1}{v}}{\Sigma \, x} \times 760. \quad \ldots \ldots \ldots \ldots \quad (4.)$$

67. If we wish to find the mean condition of the air traversed relatively to contained moisture, we evidently must not take the mean hygrometric result, but consi-

der the proportion which the masses of air obliquely traversed, and having a certain mean dampness, bear to the shorter ones with a different dose of moisture. The average dampness of the whole mass of air penetrated will be

$$\frac{\text{sum of (masses of air} \times \text{dampness of each)}}{\text{sum of masses of air}}.$$

If δ be the ratio to saturation at any hour, δ_1 the mean ratio required, and x the varying mass of air as before, we shall have

$$\delta_1 = \frac{\Sigma\, x\, \delta}{\Sigma\, x}. \quad \ldots \ldots \ldots \ldots \quad (5.)$$

68. The following Table, resuming the results of Table E, contains these quantities.

TABLE F, 25th Sept. 1832.

Apparent time.	$\frac{1}{v}$.	$\log \frac{1}{v}$.	x.	$m = \frac{\log \frac{1}{v}}{x}$.	Dampness δ.	$x\,\delta$.
h m			mm.			
8 15	1·075	·0312	450·8	·0000692	·512	230·8
9	1·141	·0573	352·8	·0001624	·530	187·0
10	1·214	·0841	286·8	2932	·489	140·2
11	1·345	·1287	255·8	5031	·484	123·8
12	1·219	·0860	245·8	3499	·452	111·1
1	1·078	·0327	254·5	1285	·451	114·8
2	1·207	·0815	284·6	2864	·526	149·7
3	1·219	·0860	350·4	2454	·617	216·2
4	1·206	·0814	500·6	1626	·620	310·4
4 30	1·321	·1209	659·5	1833	·685	451·7
Sums		·7898	3641·6	....	....	2035·7

The solution is

$$\log \frac{1}{[v]} = \frac{\cdot 7898}{3641 \cdot 6} \times 760 = \cdot 164831,$$

$$\frac{1}{[v]} = 1 \cdot 46161,$$

$$[v] = \cdot 68418,$$

Mean dampness of intercepted column δ_1 $\Big\} = \cdot 559013.$

69. Hence it appears that, *on the hypothesis of uniform opacity, a standard atmosphere of* 760$^{\text{mm}}$, *or* 29·922 *English inches of mercurial pressure, and having a mean dampness or ratio to saturation represented by* ·56 *nearly, would transmit* 68½ *per cent., or stop* 31½ *per cent. of the incident heating rays,* an estimate which agrees nearly with the mean of the results of BOUGUER and LAMBERT, and mentioned above, and very closely with the separate and indirect results obtained by Professor KÄMTZ alone on this very occasion*. This confirmation is interesting, perhaps unexpected, as the present is I believe the very first *direct* measurement of the kind.

* His estimate is ·68, not deduced from this day's observations alone, but from an extensive series.

70. Let us now examine the separate data which we have thus massed together.

71. It is evident from the equations last written, that

$$\frac{\log \frac{1}{v}}{x} = m$$

ought to be constant upon the hypothesis which we have provisionally adopted (that of uniform opacity and of uniformity of meteorological conditions). If, however, we divide the numbers in column 3 of Table F by those in column 4 (as is done in column 5), we shall find wide differences for the value of m. These may arise,—1st, from changes in the constant of opacity m, which may naturally arise from meteorological variations; 2ndly, from an error in the logarithmic hypothesis, which is founded on the physical supposition of a loss continually proportional to the intensity; 3rdly, from errors of observation. We shall consider these causes in succession.

72. I. The most important meteorological element is undoubtedly the dampness of the air; for we *know* that the formation of the slightest visible vapour instantly diminishes the solar intensity. We can hardly doubt that this action must depend upon the relative dampness of the atmosphere, that is, upon the portion of moisture existing, compared to what could exist without deposition in an equal space, and not upon the *absolute elasticity* of the vapour: for it is plain that vapour of given elasticity would make a dense visible cloud at one temperature, and might yet be compatible with intense relative dryness at another. I have therefore taken particular pains in the reduction of the hygrometric observations, and the course of the progress of dampness at Brientz and the Faulhorn in curves V. and VI. is particularly worthy of observation. At the former, the *lower* station, the dampness is greatest in the morning and evening, and has a minimum between 1 and 2 P.M. At the *upper* station, on the contrary, the dampness increases almost continually from morning till night. These facts are perfectly normal*, and are readily explained by the continual rise of the imperfectly condensed moisture which occupies the valleys of the Alps every fine night in summer, and is gradually exhaled into the upper atmosphere by the action of currents and the increasing warmth of the inferior strata,—a phenomenon from which arises (amongst other effects) the very frequent formation, about noon in the finest weather, of clouds at a height of from 8 to 15000 feet, which again give way during the advance of evening as the vapour descends.

73. The curve of mean dampness at both stations (VII.) exhibits a morning and afternoon maximum about 9 A.M. and 3 P.M., preceding somewhat the epochs of maximum loss of solar radiation already referred to. This is an important analogy, and an inspection of Curve VII. together with Curve XIV., which represents the loss of solar radiation in terms of the radiation at the upper station, will show a certain general, though not a precise analogy.

74. It cannot, however, be affirmed that these experiments are at all sufficient to show the kind of dependence which the Opacity has upon the Dampness. The values

* See DOVE's Repertorium, iv. 264.

of m, which we may call the *coefficient of extinction,* do not present any correspondence with the hygrometric variations. It is to be desired, however, that such curves should be extensively constructed.

75. II. It has been assumed that the mass of air intercepted between Brientz and the Faulhorn was equal to the differences of the barometers multiplied into the secant of the zenith distance of the sun (Art. 33.). It does not appear, however, that the ratio of heat reaching the lower station, compared with that at the upper one, varies in a geometrical progression when the thicknesses vary arithmetically. But we can hardly thence argue against the hypothesis of uniform *proportional* extinction, because the law of continuity is evidently not preserved.

76. III. Are the variations of m from hour to hour to be considered as merely the result of errors of observation? I apprehend that in some measure they may fairly be so considered, especially as resulting from a slight discontinuity in the observations at Brientz before and after one o'clock, the former being made within doors, the latter without; but the real analogy must evidently be of a somewhat complicated kind. A narrow inspection of the actinometric curves XII. and XIII., will illustrate this. It is one of the admirable results of graphical analysis, that we seize the slightest symmetry in the form of functions which might otherwise appear very dissimilar.

77. Viewed generally, we observe in these curves, *first,* that they differ from the common diurnal temperature curves (which approach more or less to the curve of sines) by drooping more rapidly at each extremity; *secondly,* that both curves have a morning and afternoon inflection before and after they attain their maximum; *thirdly,* that the curve of intensities at the upper station lies *wholly above* the curve for the lower station; *fourthly,* that the range of the former curve is greater than that of the latter; *fifthly,* that the maximum is sooner attained in the former than in the latter case.

78. Now the three geometrical characteristics last mentioned, make it plain that the *law of the differences* between the two curves must be a complex one. The analogy is very striking with the inquiry into the law of the decrement of temperature in the atmosphere at different hours and seasons, which I have fully considered in a paper in the Edinburgh Transactions*. I might, as in that case, reduce these curves to series of functions of sines, and show that the differential curve, having generally the same form, would admit of various maxima and minima in the course of the day, but I apprehend that for a single day's observations the numerical results could not have much value. I am quite confident, however, that the *five* peculiarities just mentioned of these curves will be found to be reproduced in every series made under equally favourable circumstances. If this be the case, the seeming irregularities which we are considering will be resolved into the more general consideration of the physical causes of the form of the actinometric curves. I shall make a very few remarks on each of the peculiarities above noticed.

* Vol. xiv. p. 489.

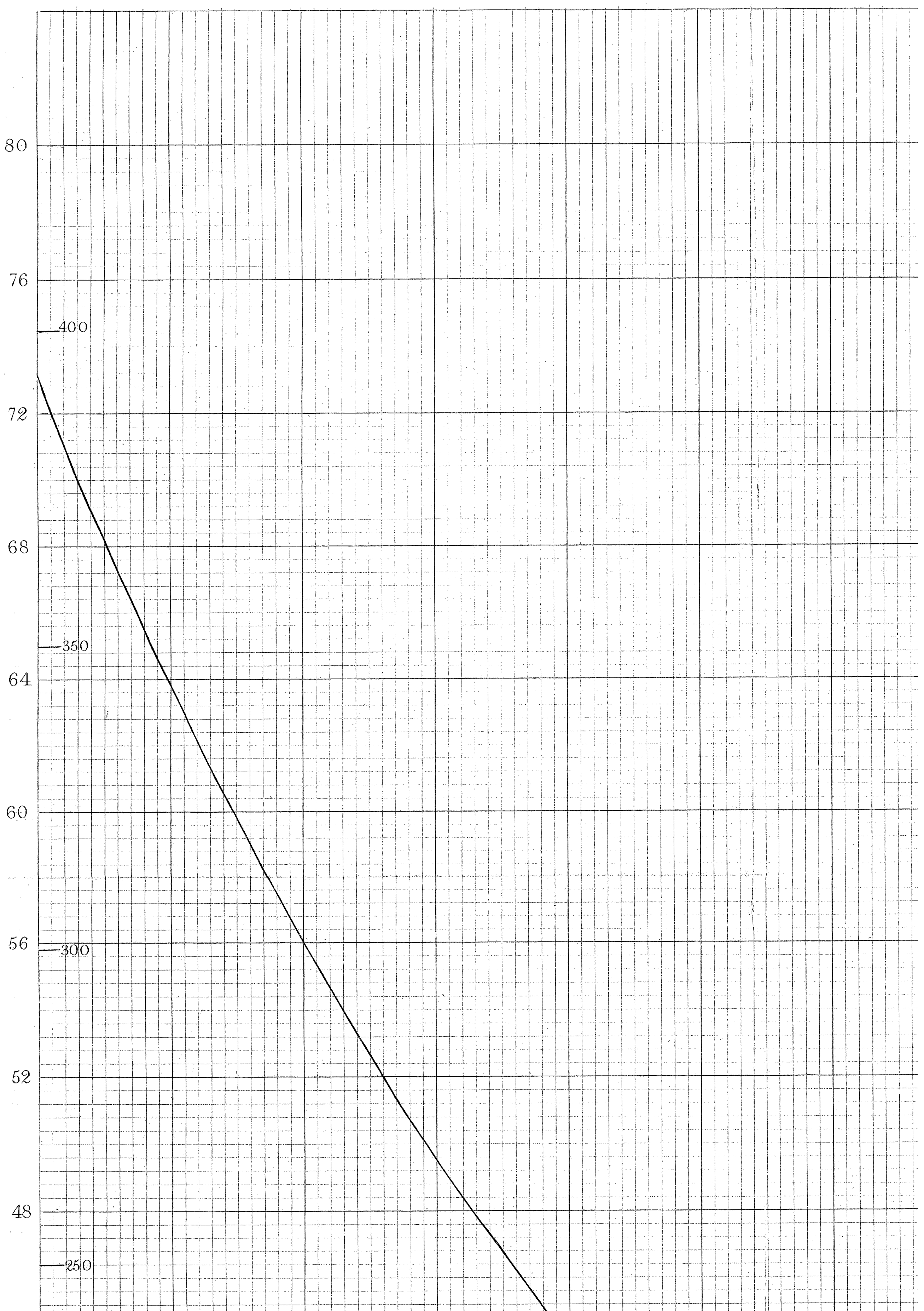
79. First. The rapid fall of the actinometric curves is due to the extreme rapidity with which the length of path increases as the sun approaches the horizon. They differ from the curve of air temperature, because that is a slow and gradual result of complicated actions ; the coolness of evening is a *continuous* result ; but the disappearance of the sun under the horizon corresponds to an instant extinction of the force of radiation. Secondly. With respect to the points of contrary flexure which occur about two hours before and after their maximum values, and which in both curves are slightly marked in the morning, and more intensely in the afternoon, they probably arise from the combination of a two-fold effect of the sun's elevation. The one is the increased intensity as the sun is higher, the other is the transference of vapour from the lower to the higher regions of the air by the heating of the lower strata, producing the incipient condensation at a certain elevation, already alluded to as the cause of the slight clouds which often appear between ten and twelve o'clock. As the sun's power diminishes, and the vapours redescend into the less rarefied and warmer regions, they are in some degree redissolved in the afternoon, and the increased transparency of the atmosphere (which will besides be aided by the general maximum of the temperature of the air occurring in the plains between two and three o'clock, and producing also a maximum of dryness there) checks the downward progress of the curve due to the increasing obliquity of the rays. Thirdly. The curve at the upper station lies wholly above that at the lower station, on account of the absorption of heat in every case by the intercepted air. Fourthly. The range at the higher station is greater than that at the lower. This is an evident and necessary consequence of the fact, that the maximum above must exceed the maximum below, and that at sunset and sunrise they must both pass through zero. It might be more correct, however, to consider the continuous part of the curve extending to the moment *before* sunset from the moment *after* sunrise. In this case we might expect the difference of intensity at the two stations to increase very rapidly with the obliquity of the sun's rays, so that the two curves, instead of approaching one another in the morning and evening as they appear to do (Curves XII. and XIII.), ought to separate further. It is to be recollected, however, that the extinction in any stratum varies with the intensity of the *incident* heat, and that being very small near the horizon, the absolute extinction will be very small also. Nevertheless, the *relative* extinction may be very great, as appears from the form of the right-hand branch of Curve XIV. The morning branch does not show the same effect, and this may be thus explained. The evening vapours are dense and absorptive ; in the morning the atmosphere is comparatively clear, especially amongst mountains. To this circumstance must be imputed the much more rapid fall of the Curves XII. and XIII. in their evening than their morning branch. But further, it will be shown presently that the law of uniformly regular extinction is not true, and that the loss in passing through a medium, is not only absolutely but relatively (to the intensity) greater at first than afterwards: that when the thicknesses are very great, any additional thickness intercepts but little of the radiant force; conse-

quently, near the horizon, a great thickness of atmosphere having been traversed by the rays which reach the upper station, even the obliquity of the passage to the lower station does not (unless the inferior strata be particularly loaded with vapours), cut off anything like a corresponding portion of solar heat, and a second equal mass would intercept still less.

80. Fifthly, and lastly, the maximum of intensity is sooner attained above than below. This arises, no doubt, mainly from the fact (amply confirmed by the hygrometric curves), that the sun shines with a disproportionate intensity during the morning on the upper station, owing to the mass of vapours being then in the valleys. The solar intensity will therefore attain an earlier maximum, since after ten or eleven o'clock a quantity of vapour rises between the upper station and the limit of the atmosphere, and therefore throws the maximum rather before noon. In the plains, on the contrary, where the *whole* atmosphere is *all day* between the observer and the sun, the maximum will incline towards the period of maximum dryness of the day, that is, it may be an hour or half an hour after noon. The curve of mean dampness VII., with its point of contrary flexure in the afternoon, entirely confirms this view, and the diurnal curve of temperature at Brientz, marked I., shows both inflections in the clearest manner.

81. From the comparison of the two curves of solar intensity, we have deduced the mean loss of heat intercepted between the two stations, and we have thence concluded that, on the hypothesis of uniform opacity, about *one-third* of the solar heat is lost by vertical transmission through the atmosphere. It is interesting to compare this result with that which is deducible from the individual observations at either station. In that mode of viewing the subject, it appears from BOUGUER's reasoning (see Art. 7), that two observations at different altitudes are, in point of rigour, sufficient for deducing the loss due to vertical transmission. If more than two values have been got, they may be combined in two series of which the means are taken; or they may be treated by the method of least squares, which will give the most probable result, on the hypothesis of the diminishing geometrical progression of the intensities. It is more interesting and important, however, to employ the superfluity of observations in testing the accuracy of the assumed law, rather than in giving a merely illusory degree of precision to the results of a law which may be wrong. For this purpose I projected, by rectangular coordinates, the intensities observed, the thicknesses of homogeneous air traversed (computed from the sun's altitude) being the horizontal coordinate or independent variable. On the law commonly assumed, the points thus determined ought to lie in a regular logarithmic curve, which being readily prolonged by geometry or by calculation, would give the intensity corresponding to thickness 0, or the degrees which the actinometer B. 2. would show if placed *wholly beyond the atmosphere*.

82. When the projection came to be made, I remarked, with much interest and some surprise, the admirable agreement between the insulated observations at both



80
76
400
72
68
350
64
60
56
300
52
48
250

Nº. XV

INTENSITY OF SOLAR RADIATION,

ON THE SCALE OF ACTINOMETER B. 2. — ON THE 25TH. SEPTR. 18

AT BRIENTZ & THE FAULHORN.

T^{R} 1832.

BRIENTZ

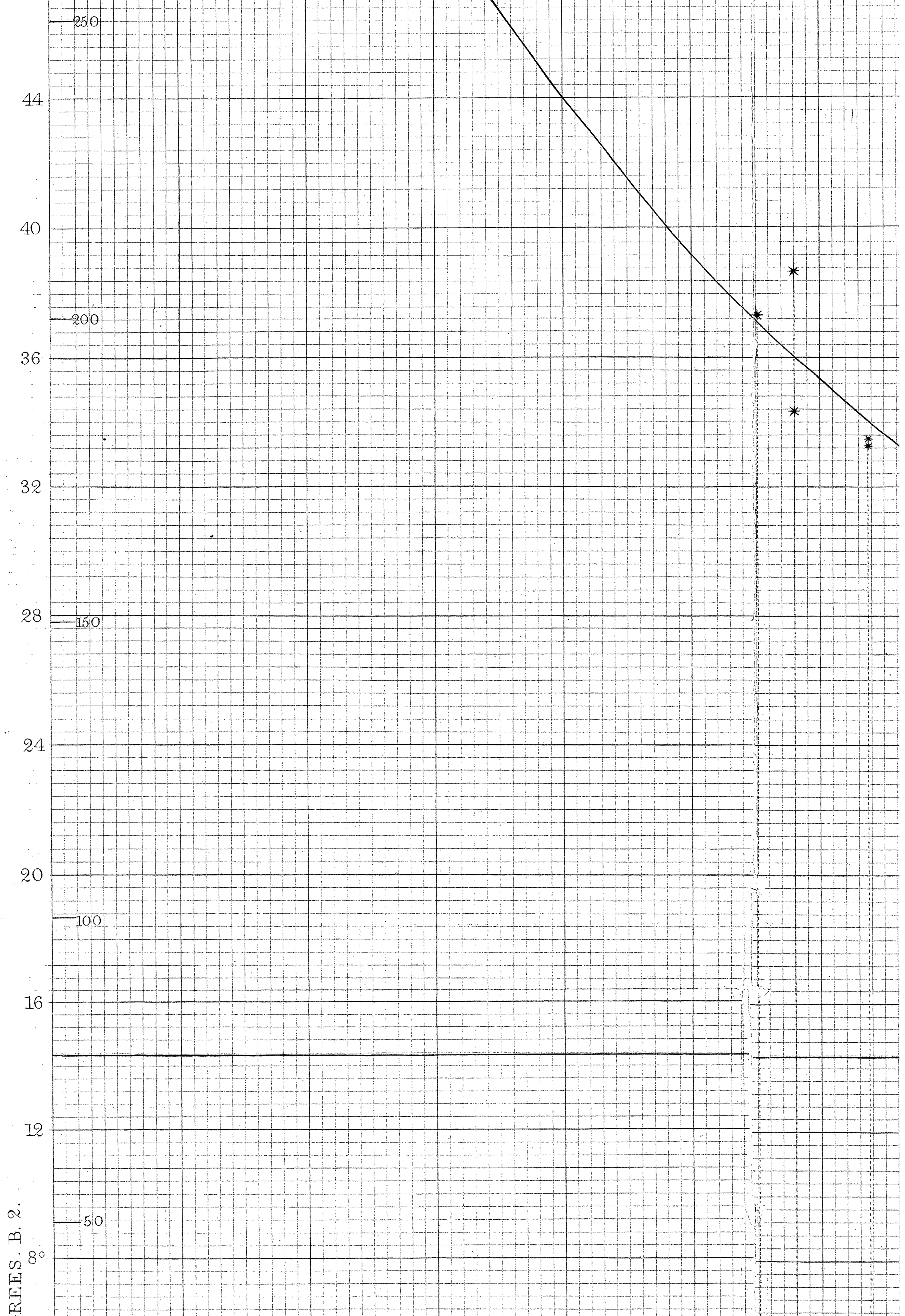
Hour App^{t} Time	Thickness Atmosphere *Millimetres*	Actinometer
$8\frac{1}{4}$	1952	22·8
9	1529	25·5
10	1244	27·6
11	1113	28·7
12	1073	30·6
1	1111	31·8
2	1242	27·6
3	1529	24·2
4	2184	19·9
$4\frac{1}{2}$	2874	16·2

The observations ...
by full lines and ...
Faulhorn by dotted ...

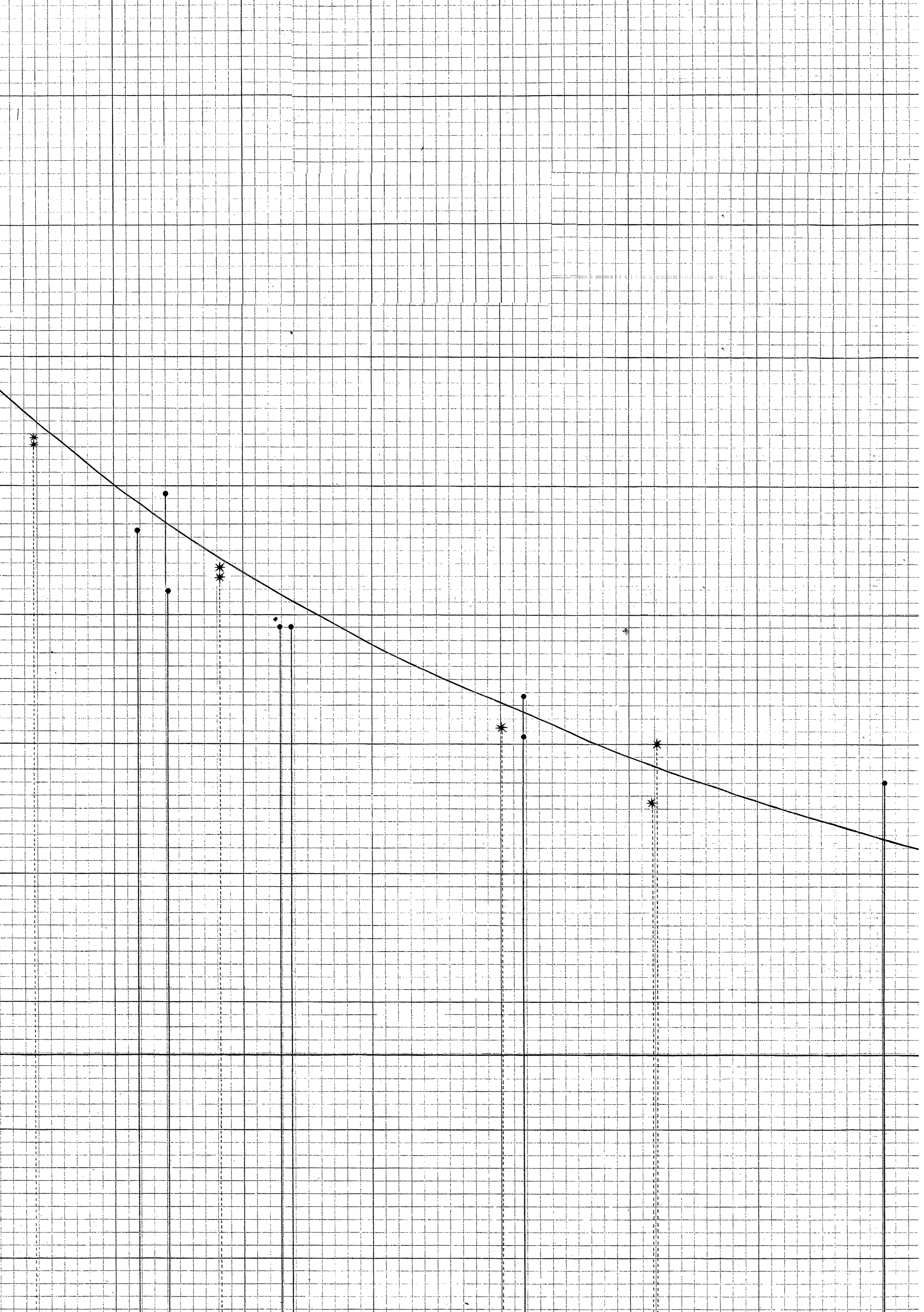
FAULHORN

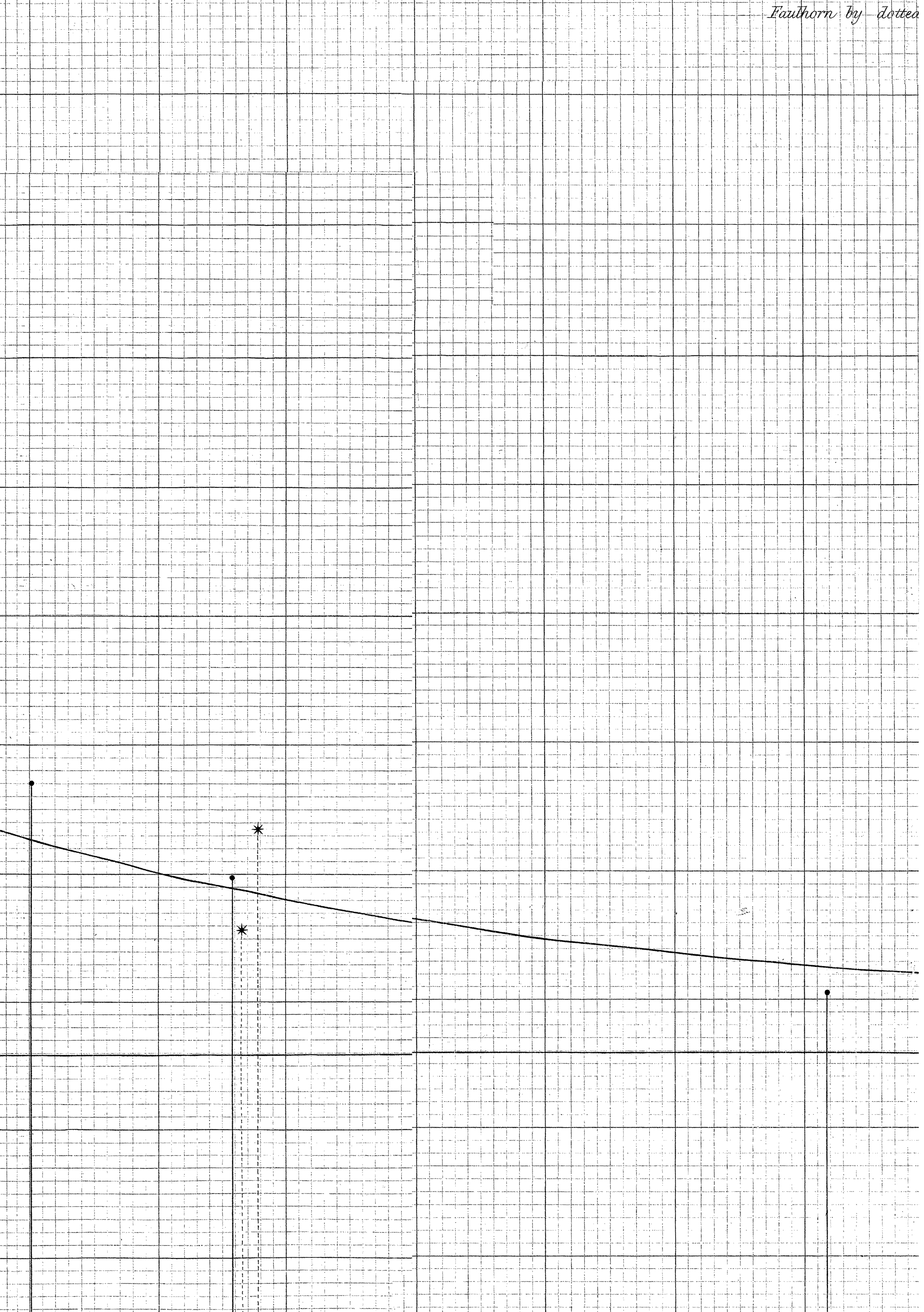
ometer	Hour App^t. Time	Thickness Atmosphere *Millimetres*	Actinometer
	7½	2197	18·2
	8	1676	22·2
2·8	8¼	1501	24·5
5·5	9	1176	29·1
7·6	10	957	33·5
3·7	11	857	38·6
9·6	12	827	37·3
1·8	1	857	34·3
7·6	2	958	33·3
4·2	3	1178	29·5
9·9	4	1683	24·0
6·2	4½	2214	21·4

…vations at Brientz are distinguished
…nes and black dots, those at the
…y dotted lines and stars.

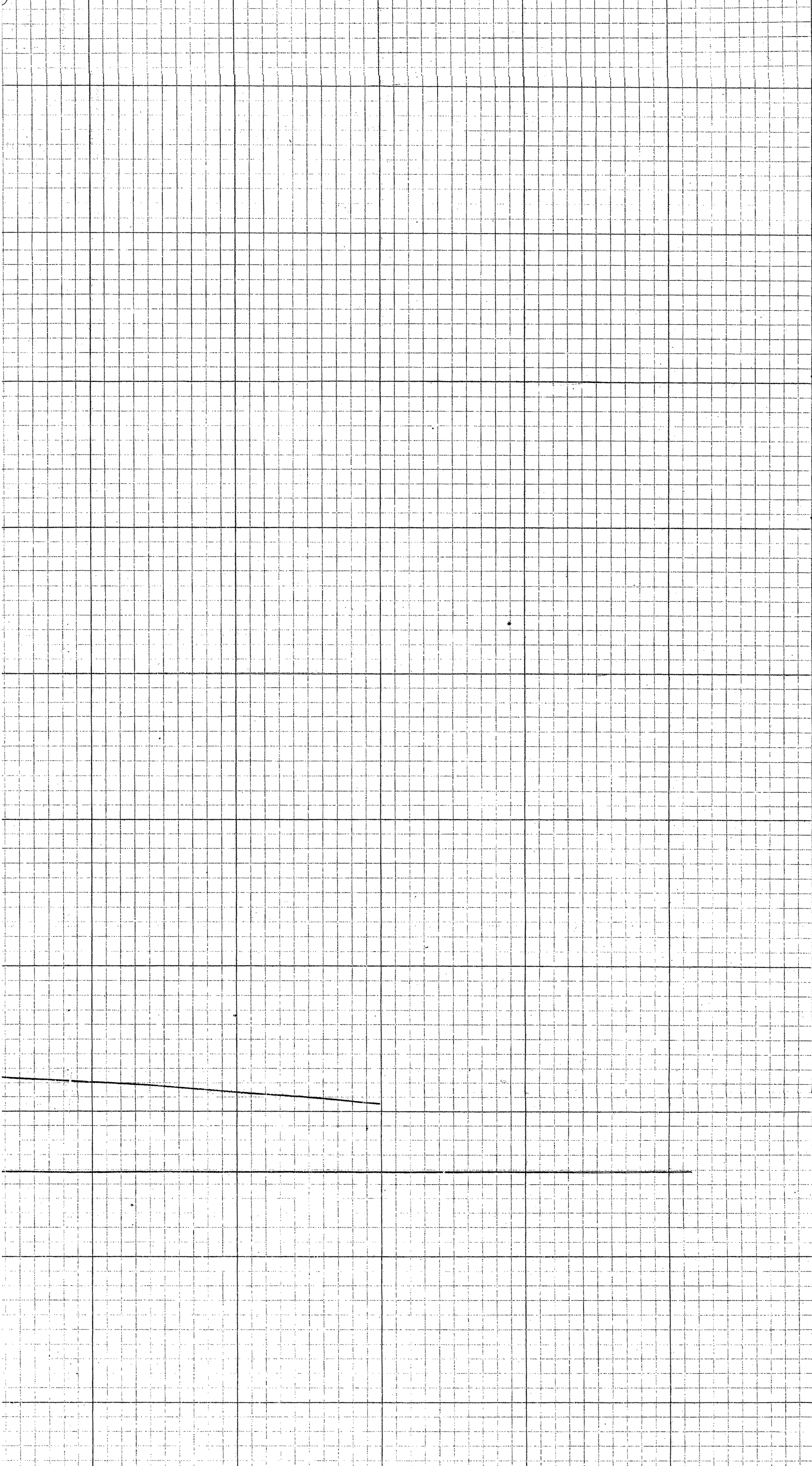


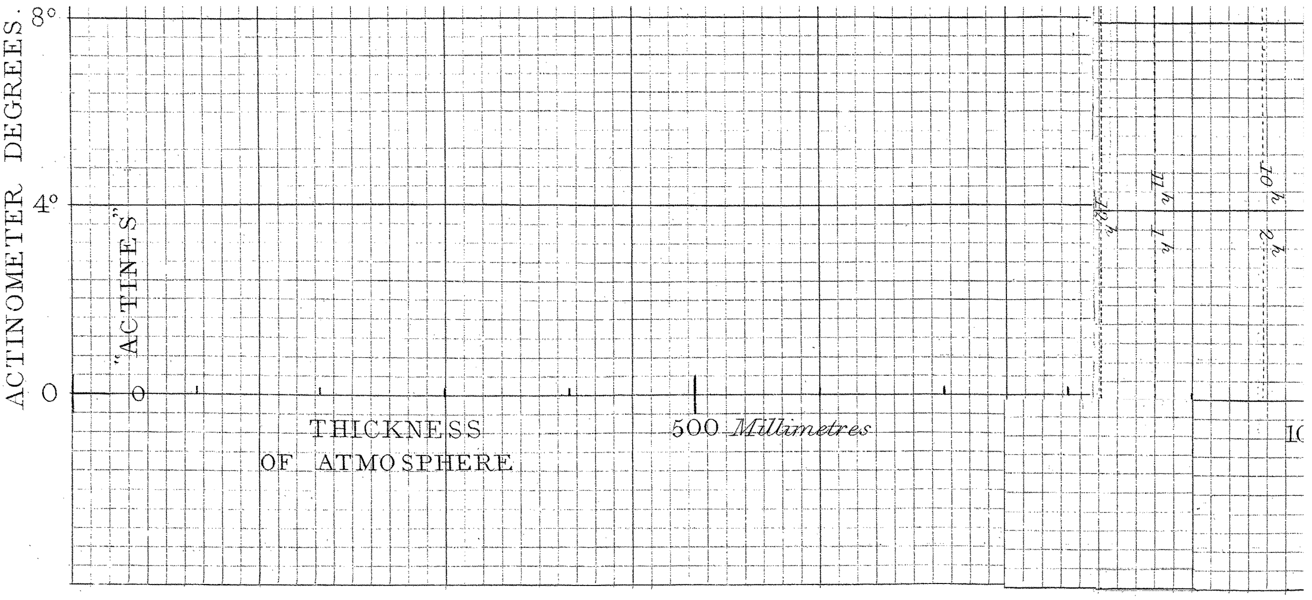
250
44
40
200
36
32
28
150
24
20
100
16
12
50
8°
REES. B. 2.



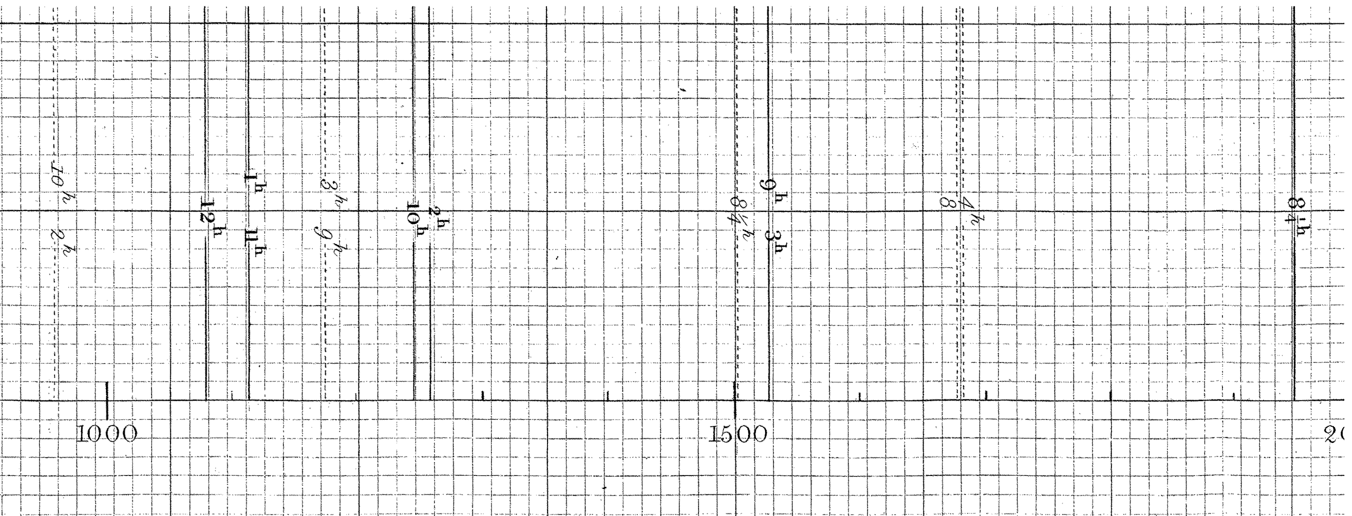

Faulhorn by dotted

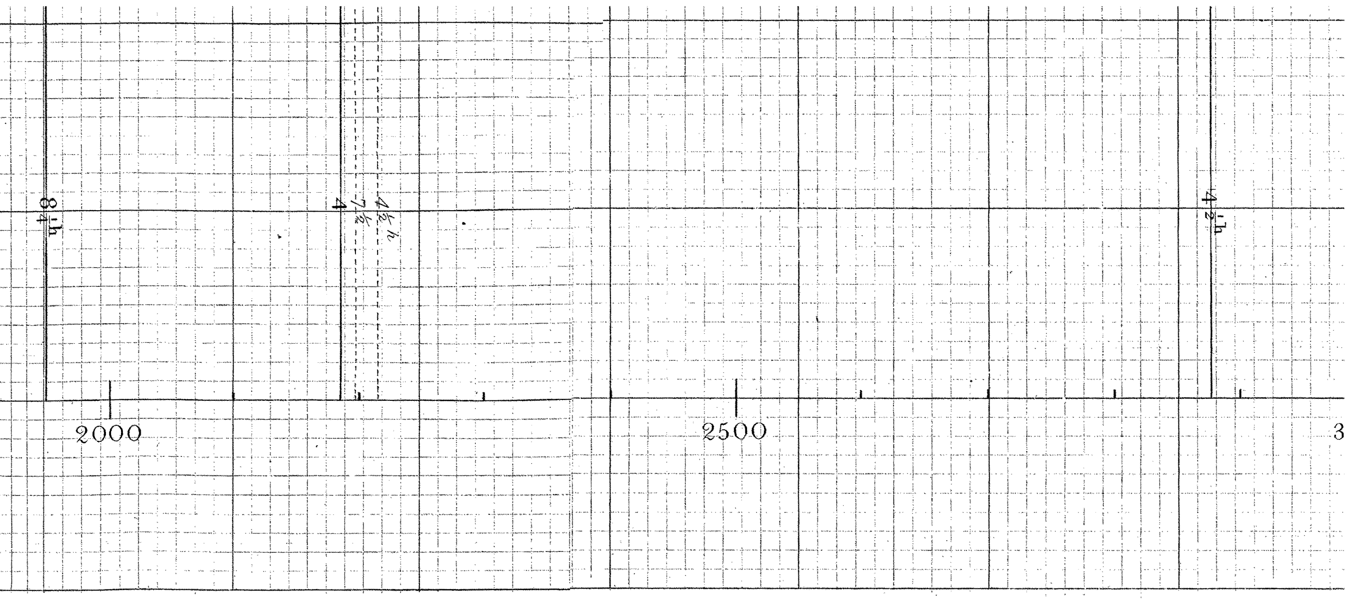
y dotted lines and stars.



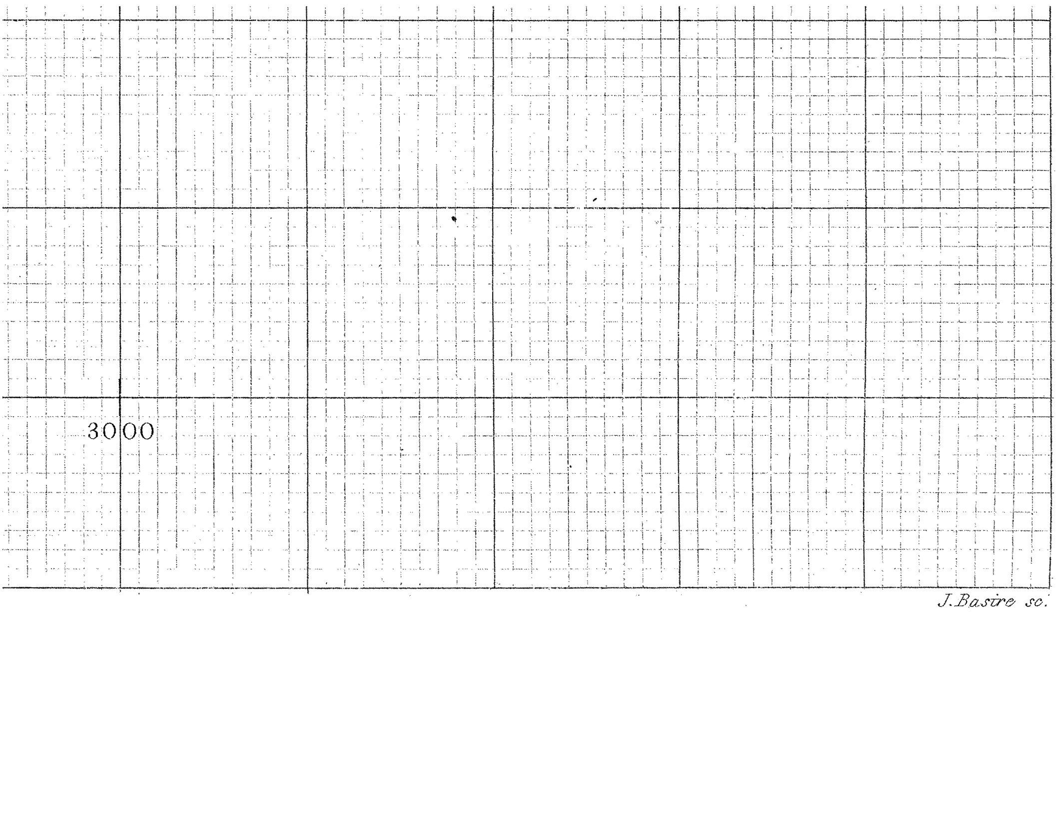


ACTINOMETER DEGREES.
8°
4°
0
"ACTINES"
0
THICKNESS
OF ATMOSPHERE
500 Millimetres
11 h
1 h
10 h
2 h


1000
1500



8¼ h
4
7½
4½ h
2000
2500
4½ h



J. Basire sc.

stations. Not only did the *continuity* of the law which both series followed prove the exactness of the reduction of intensities obtained with one instrument into degrees of the other, but what I have called *unexpected*, was the fact that an equal ordinate or intensity should be indicated for a passage through an equal *mass* of air at both stations. For that mass of air, it is to be observed, was very differently composed in the two cases. On the Faulhorn, a very oblique transit through the rare air, superior to 8400 feet, was requisite to give the same mass as a less oblique transit through the whole atmosphere, in order to arrive at the lower station. It is very far from evident that an equal loss should take place in both cases: yet when the observations were projected in the form of Curve XV. Plate XXII., without regard to the station at which they were made, they were found to range perfectly well together, so that one and the same interpolating curve passed naturally and easily through either series, or through both.

83. I first sketched by the eye, and without respect to any theory, a curve which appeared to satisfy the observations of the 25th of September, which curve, it is to be observed, was to give the law of extinction of heat in the atmosphere, and by its arbitrary prolongation, to assign the solar intensity beyond the atmosphere, and the absorption due to a vertical transmission.

Sect. VI.—*Concerning the Law of Extinction.*

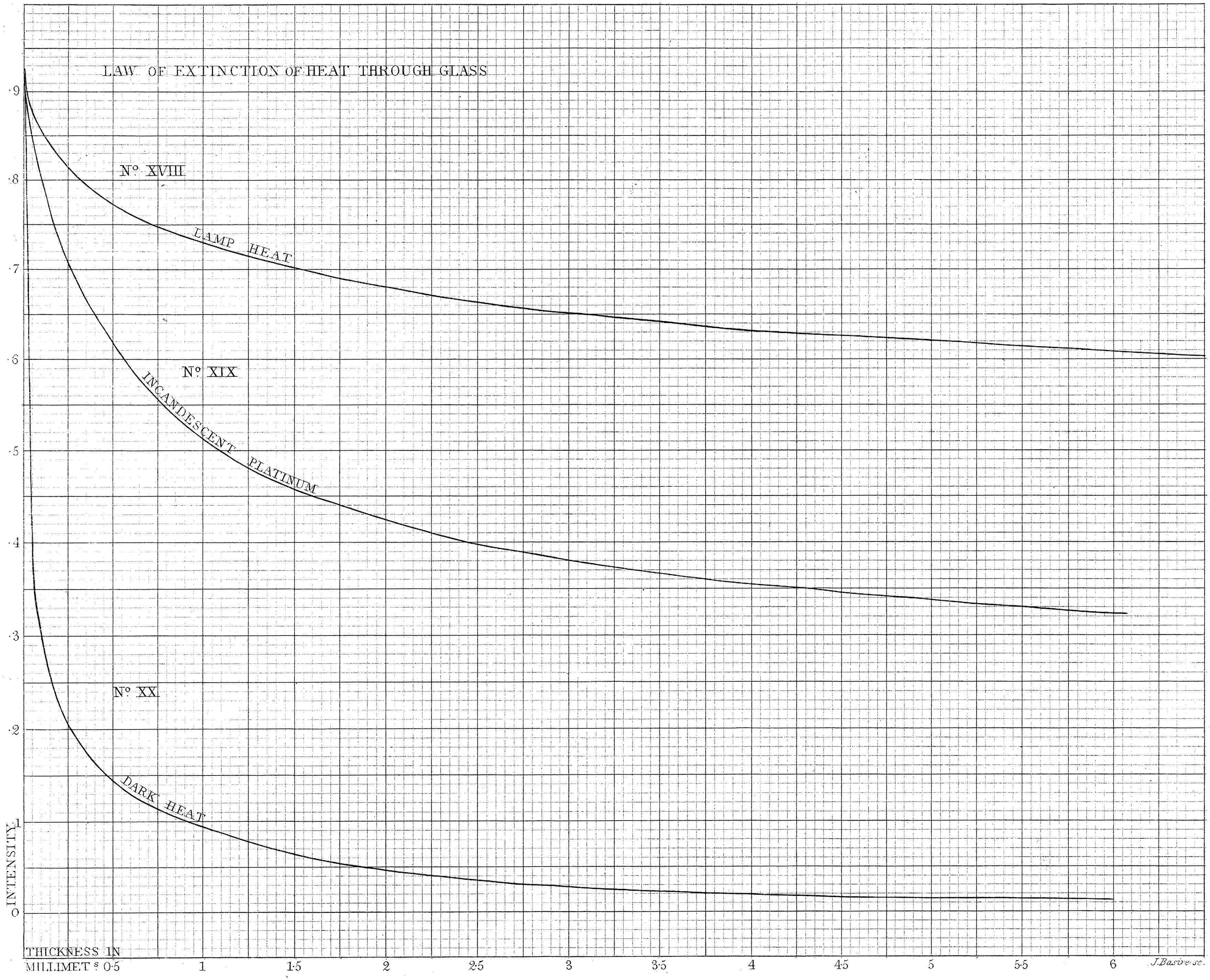
84. Many familiar facts connected with the extinction of heat and light in passing through media, some of which have been adverted to in the earlier part of this paper, render it very unlikely that the part of the solar rays which affects the actinometer should suffer a *uniform relative* loss in the successive strata of air. Perhaps no medium whatever merely extinguishes light without colouring it, and if it colours it, the light, being first deprived of those portions or rays for which the medium in question is comparatively opake, will be more and more freely transmitted through similar successive obstacles. We have seen (Art. 13.) how Lambert established the law of the progressively-increasing diathermancy of successive plates of glass, a result confirmed by De la Roche and Melloni: and as we have found that, notwithstanding the rarity of the atmosphere, its resistance to the passage of even the solar rays is considerable, it is a most probable thing that a similar law should hold in that case. A very slight inspection indeed of Curve XV. shows that it rises much faster than in a simple geometrical progression. What law will best satisfy the observations? and how far are we justified in pushing it beyond the limits of experience, as for instance, to the surface of the atmosphere?

85. The safest induction would appear to be by endeavouring to generalize the *law of extinction* of heat in various media. But here we are met by peculiar difficulties. A certain number of experiments have been made on the extinction of heat from terrestrial sources in passing through different thicknesses of media, such as glass, rock crystal, or water. I allude particularly to those made by M. Melloni,

at the suggestion of M. BIOT, of which the latter has given an elaborate analysis*. Now when the curves of intensity of transmitted heat are projected in terms of the thickness of the transmitting medium, it appears that the rate of extinction is much more rapid at first, becomes continually slower, and long before the curve has reached the axis, or the heat has been wholly absorbed, it runs parallel to the axis without ever approaching it; in other words, it has an asymptote parallel to, but at a distance from the axis. This corresponds to the physical fact, that when heat has been already transmitted through a great thickness of any medium, provided it be mechanically pure, an increase of thickness will produce little or no extinction.

86. The cases of extinction which I have most narrowly considered are those of lamp-heat, heat from incandescent platinum, and dark heat, through glass. These curves are projected in Plate XXIV., the ordinates representing the intensities of heat transmitted at different thicknesses, the incident heat being unity, but which is reduced to ·925 according to MELLONI, by reflection at the two bounding surfaces. The existence of an asymptote or final value of the transmitted heat in every one of these cases is abundantly evident, and this would be one of the constants (variable for different media) which would determine the equations to the curves, which might be expected to be of one species. There is, however, the utmost difficulty in representing these laws of extinction by one tolerably simple *continuous* form; and however desirable it may be that such a form should be discovered, so that a portion of the system of ordinates being found, the remainder may be deduced, we must admit that there is little physical probability for its permanence. And for this reason: the incident rays may be imagined to be composed of a great, but definite number of portions of radiant matter, of distinct qualities as regards the rate of extinction. We may suppose, for simplicity, that each individual homogeneous ray (or congeries of similar rays) is extinguished according to the simple logarithmic law. But each ray has its own modulus, or coefficient of extinction, which depends on two things, namely, the composition of the incident heat, and the specific nature of the medium, as regards each of the integral kinds of heat. Hence the *initial* rate of extinction will depend almost entirely upon the portion of heat very easily extinguishable, which exists in the calorific beam, and not sensibly upon those *persistent* kinds of heat for which the medium in question is nearly diaphanous; whilst at great thicknesses the former class of rays being entirely extinguished as to sense, the latter class, namely, the more persistent ones, alone exercise any influence on the curve of extinction. Thus it appears, that since we have no *à priori* method of discovering the composition of any mixed kind of heat from such a source as the sun, it must be *impossible* to conclude with certainty the law of loss or extinction at *small* thicknesses, from observations of the law of extinction at *great* thicknesses; for they are not in point of fact the same rays which are undergoing extinction in the one case and in the other, and therefore the continuity of the law cannot be assumed with any degree of certainty. The indication of

* Mémoires de l'Academie des Sciences, tom. xiv. p. 493, &c. (printed in 1838).


LAW OF EXTINCTION OF HEAT THROUGH GLASS
Nº XVIII
LAMP HEAT
Nº XIX
INCANDESCENT PLATINUM
Nº XX
DARK HEAT
INTENSITY
·9
·8
·7
·6
·5
·4
·3
·2
·1
0
THICKNESS IN MILLIMETˢ 0·5
1
1·5
2
2·5
3
3·5
4
4·5
5
5·5
6
J. Basire sc.

the true law could only, in fact, arise from the minute residual quantity of the more extinguishable rays existing at great thicknesses; quantities so small that the law of their variation would be lost in the errors of observation.

87. The analogy of the case of light will perhaps illustrate this important consideration. Suppose solar light to be incident upon intensely red glass: at very minute thicknesses some part of every kind of light will, no doubt, pass through, but we know that the old veneered homogeneous glass transmits pure red light, even when it is very thin indeed. At still greater thicknesses only red light will be transmitted, and that with as great freedom, perhaps, as common window-glass permits the passage of white light. The intensity then of the red light, for which the glass is perfectly transparent, indicates the *residual* constant quantity, towards which the transmitted beam continually approximates, and which is very far from zero of intensity. But it is evident that however numerous and complete our observations upon the law of absorption of light (without respect to colour) might be at all but the least thicknesses of such red glass, it would be impossible to deduce from them alone the law of extinction of all those kinds of light for which the medium in question is very nearly opake, as, for instance, the yellow or the blue, and consequently it would be impossible even to approximate to the primitive intensity of the compound incident beam.

88. M. Biot, in the memoir already referred to on M. Melloni's experiments, is so sensible of these difficulties, that he has contented himself with arbitrarily dividing the incident heat into three kinds or qualities as respects extinction, calculating by a separate law for each, and assuming the sum as the representation of the transmitted heat; a process by which, no doubt, almost any series of facts might be represented, and which therefore gives very little information as to the real law of extinction.

89. I have spent much labour on the same subject, of which it would be out of place here to detail the results. I have indeed obtained a form which contains only three constants, and which expresses tolerably the law of extinction of heat in solid media. But this investigation satisfied me that where the medium is so very absorptive as most solids are, it is wholly impossible to deduce the form of the curve near its origin, from the remoter portion of it.

90. However desirable it might be to proceed by the direct analogy of media, for which we may ascertain the law of extinction, to that of the atmosphere, in which we can only ascertain it for certain considerable thicknesses, the circumstances now detailed appear to render an investigation of such generality entirely useless. In the discussion of the Curve XV. I have thrown aside every other consideration, and attempted to obtain an empirical formula which shall satisfy the law of extinction within the very considerable limits of thickness observed on the 25th of September, 1832, viz. from 827 millimetres to 2874 millimetres of mercury for the equiponderant column.

91. A curve, nearly approaching to that of Curve XV., has been drawn through the points therein defined, which curve, as already stated, was drawn by the eye without reference to any theory whatever. Tangents were *mechanically* drawn to this curve, and the *rate of loss* for a given thickness was ascertained corresponding to equal increments of intensity, or for the parallels of 15, 20, 25, 30, 35, 40, and 45 actinometric degrees. For convenience, the rate of loss was found corresponding to 500 millimetres of mass of air in each case, and found to be

$$1^{\circ}\cdot2 \quad 3^{\circ}\cdot0 \quad 6^{\circ}\cdot0 \quad 9^{\circ}\cdot5 \quad 12^{\circ}\cdot8 \quad 15^{\circ}\cdot7 \quad 18^{\circ}\cdot3,$$

numbers nearly in arithmetical progression.

92. The rate of loss in terms of the intensity is projected in Curve XVI., Plate XXIII., and an interpolating straight line has been passed through the points. Now, were the loss everywhere proportional to the intensity, which is the logarithmic law, the straight line would have cut the axis when the intensity = 0, whereas the rate of loss vanishes when the intensity = 14°·3 nearly, which indicates the limit towards which the intensity is continually tending, below which it cannot fall, and which is consequently the position of the asymptote. The equation to this line is of the form

$$x = a\,v - b,$$

where v is the intensity in actinometric degrees, and x being the rate of extinction for *one* millimetre of thickness, $a = \cdot001224$, $b = \cdot0175032$, whence the rate of extinction for 500 millimetres has been computed, in order to be compared with the graphical results (of which the possible errors, as in every case of drawing tangents, are very sensible).

Intensity.	Rate of loss for 500 millimetres. Observed.	Calculation.	Differences.
45°	18°·3	18°·79	+0°·49
40	15·7	15·73	+0·03
35	12·8	12·67	−0·13
30	9·5	9·61	+0·11
25	6·0	6·55	+0·55
20	3·0	3·49	+0·49
15	1·2	0·42	−0·78

93. Assuming the form of this approximation to be satisfactory (with a slight modification of the constants), we have for the value of the first differential coefficient of the equation to the Curve XV.,

$$-\frac{d\,v}{d\,x} = a\,v - b \quad \ldots \ldots \quad (a.)$$

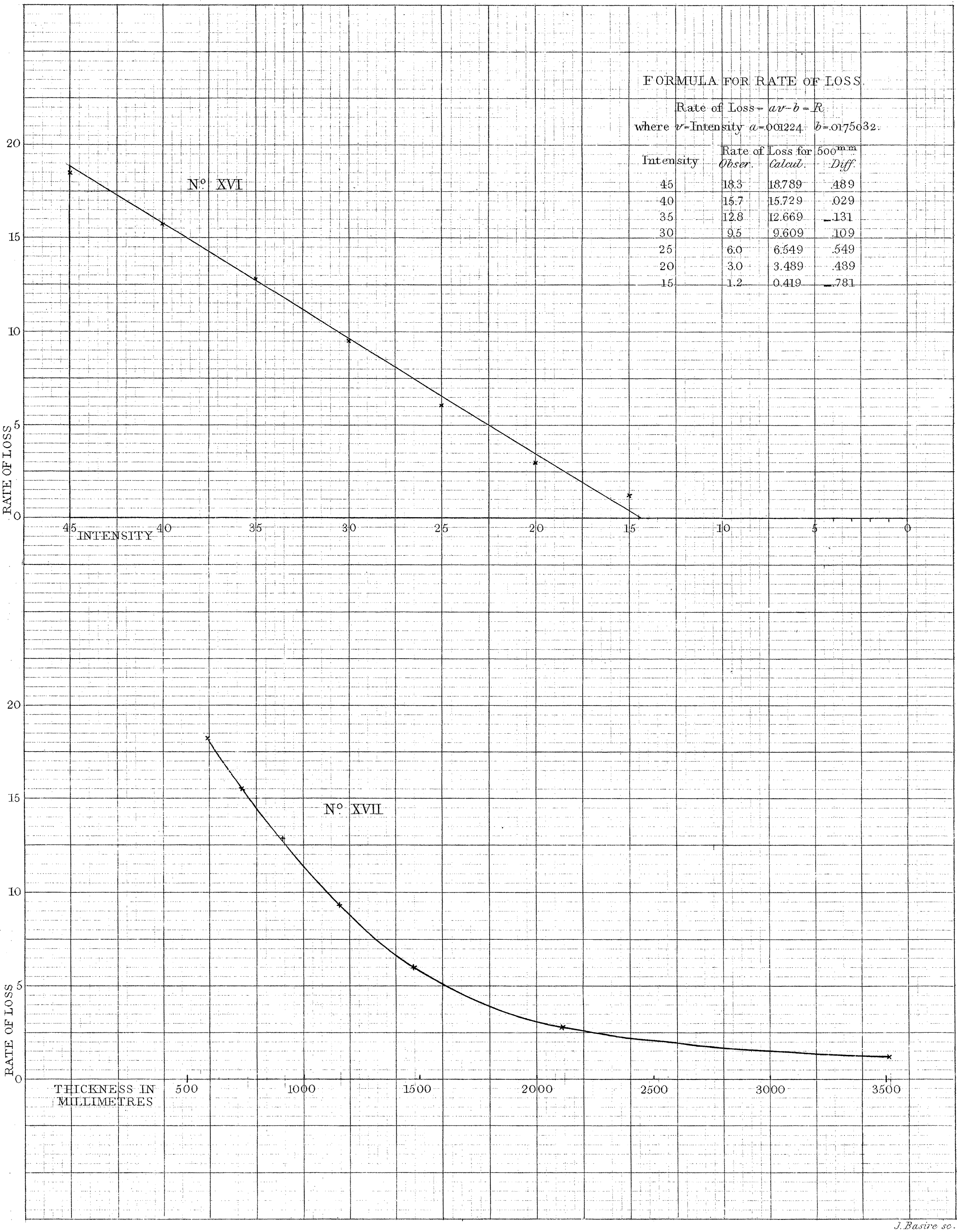
$$d\,x = -\frac{d\,v}{a\,v - b}.$$

$$x = -\frac{1}{a}\log\,(a\,v - b) + c \quad \ldots \ldots \quad (b.)$$

$$= -\frac{1}{a}\log\left(\frac{d\,v}{d\,x}\right) + c \quad \ldots \ldots \quad (c.)$$

LAW OF ABSORPTION OF LIGHT THROUGH THE ATMOSPHERE.

25th Septr 1832.



FORMULA FOR RATE OF LOSS.

Rate of Loss $= av - b = R$

where v = Intensity $a = .001224$ $b = .0175032$.

Intensity	Rate of Loss for 500mm *Obser.*	*Calcul.*	*Diff.*
45	18.3	18.789	.489
40	15.7	15.729	.029
35	12.8	12.669	−.131
30	9.5	9.609	.109
25	6.0	6.549	.549
20	3.0	3.489	.489
15	1.2	0.419	−.781

J. Basire sc.

Whence, if the rate of loss $\frac{dv}{dx}$ be projected in terms of the thickness, as in Curve XVII., it ought to give the logarithmic curve, which it evidently approaches nearly.

94. When $x = 0$, let $v = \mathrm{V}$ the intensity in actinometric degrees exterior to the atmosphere,

$$x = \frac{1}{a} \log \frac{a\mathrm{V} - b}{av - b}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (d.)$$

When $a v = b$, or $v = \frac{b}{a}$, $x = \infty$, $\frac{b}{a}$ is therefore the distance from the horizontal axis to the asymptote.

95. Dividing both numerator and denominator in equation ($d.$) by a, it becomes

$$x = \frac{1}{a} \log \frac{\mathrm{V} - \frac{b}{a}}{v - \frac{b}{a}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (e.)$$

which is the equation to a logarithmic curve whose general ordinate is $v - \frac{b}{a}$ instead of v. This, therefore, is the form of the curve of extinction in Curve XV.

96. For calculation, (the logarithms being hyperbolic, and ε denoting the base of that system) by equation ($d.$),

$$\varepsilon^{ax} = \frac{a\mathrm{V} - b}{av - b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (f.)$$

and any corresponding ordinates v and x being given, as well as the values of a and b, we may deduce the initial value or V in the following terms:—

$$\mathrm{V} = \frac{1}{a} \{b + (av - b)\,\varepsilon^{ax}\}\,; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (g.)$$

and taking Tabular Logarithms,

$$\log\left(\mathrm{V} - \frac{b}{a}\right) = \log\left(v - \frac{b}{a}\right) + a\,x \log \varepsilon. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (h.)$$

97. In the construction of the Curve XV., constants a little different from those above found have been used as expressing the *mass* of the observations rather better. The value of $\frac{b}{a}$ instead of 14°·3 has been assumed at 15°·2. A line is drawn parallel to the axis of x at that distance, and a logarithmic curve constructed upon it, with the value of m in equation (1.) Art. 7, which is the same as $a \log \varepsilon$ of equation (h), equal to

·00050708.

The following ordinates have been thence computed.

Thickness, or x in millimetres of mercury.	Intensity, or v in degrees of Actinometer B. 2.
0	57°·86 + 15°·2 = 73°·06
500	32·3 + 15·2 = 47·5
1000	18·0 + 15·2 = 33·2
1500	10·3 + 15·2 = 25·5
2000	5·6 + 15·2 = 20·8

Thickness, or x in millimetres of mercury.	Intensity, or v in degrees of Actinometer B. 2.
2500	$3°\cdot1 + 15°\cdot2 = 18°\cdot3$
3000	$1\cdot7 + 15\cdot2 = 16\cdot9$
infinite	$0\cdot0 + 15\cdot2 = 15\cdot2$

Hence, supposing the approximation to the initial intensity of the solar heat to be sufficient, the portion transmitted through an atmosphere balanced by 760 millimetres of mercury will be found to be

$$\frac{39\cdot03}{73\cdot06} = \cdot534 \text{ of its whole amount.}$$

The value of V is by no means given as certain: it may very probably be *greater*, even *much greater* than has been assigned, but it is very unlikely to be *less*.

98. Hence, too, the absolute intensity of the solar ray has been very much underrated by all writers. The portion vertically transmitted probably does not exceed a half, instead of being equal to two-thirds or three-quarters, as has generally been supposed. BOUGUER's estimate for light approaches nearest to it, but that was founded on the logarithmic law, which we have shown not to be applicable, at least for heat.

99. It may be interesting, however, before finally quitting the observations of the 25th of September, 1832, to inquire what results we should have deduced from them upon the old hypothesis of the intensity diminishing in geometrical progression, and thus to render the observations directly comparable with those of BOUGUER, LAMBERT, LESLIE, KÄMTZ, and POUILLET, that is, so far as I am aware, of every author who has published any determination of the opacity of the atmosphere, including LAPLACE.

100. Resuming the notation of Art. 7, which we used to describe BOUGUER's method, where v_1 and v_2 are two intensities expressed in actinometric degrees, x_1 and x_2 the corresponding atmospheric masses traversed expressed in millimetres of mercury. By equation (3.) of that article* we find the value of the coefficient of extinction

$$m = \frac{\log \frac{v_2}{v_1}}{x_1 - x_2}.$$

And if $[v]$ = the intensity after a vertical transit through the atmosphere, the intensity beyond the atmosphere being = 1, we have by equation (1.) of that article,

$$\log \frac{1}{[v]} = m \times 760.$$

When more than two values of v and x are used we may divide them into two series, and take the arithmetical means of log v and x for each.

101. Now a good deal depends upon the way in which these series are formed. We may combine the observations, so that the observations on the shortest atmospheric columns forming one series shall be set against those of the longest columns in another. Thus the observations at Brientz alone give the following results.

* It will be seen that this equation is obtained in exactly the same way as that of Art. 65.

Brientz.

	First Series.			Second Series.	
Hour.	Log *v*.	*x*.	Hour.	Log *v*.	*x*.
$8\frac{1}{4}$	1·3579	1952	10	1·4409	1244
9	1·4065	1529	11	1·4579	1113
3	1·3838	1529	12	1·4857	1073
4	1·2989	2184	1	1·5024	1111
$4\frac{1}{2}$	1·2095	2874	2	1·4409	1242
Sum.....	6·6566	10068	Sum.....	7·3278	5783
Mean....	1·3313	2014	Mean....	1·4656	1157

Taking $V = 1$, $[v] = \cdot7602$, $m = \cdot0001567$
V expressed in actinometric degrees (B. 2.) = 44·35.

Faulhorn.

	First Series.			Second Series.	
Hour.	Log *v*.	*x*.	Hour.	Log *v*.	*x*.
$7\frac{1}{2}$	1·2601	2197	10	1·5250	957
$8\frac{1}{4}$	1·3892	1501	11	1·5866	857
9	1·4639	1176	12	1·5717	827
3	1·4698	1178	1	1·5353	857
4	1·3802	1683	2	1·5224	958
$4\frac{1}{2}$	1·3304	2214			
Sum.....	8·2936	9949	Sum.....	7·7410	4456
Mean....	1·3823	1658	Mean....	1·5482	891

$V = 1$, $[v] = \cdot6848$, $m = \cdot0002163$
V expressed in actinometric degrees (B. 2.) = 55·07.

102. We thus find that when the *extreme* observations of each series are employed, the Faulhorn observations give a greater intensity to the extra-atmospheric radiation, and consequently a greater coefficient of extinction to the atmosphere, because the part of Curve XV. corresponding to the least thickness rises proportionably faster than the other part. But both results give greatly inferior extra-atmospheric radiation than the corrected hypothesis we have assumed; the first set gives 44°, the second 55°, the corrected hypothesis as much as 73° for the extra-atmospheric radiation.

103. If we avoid *extreme* columns and arrange the observations in alternating series so as to present a feeble but well-ascertained mean difference, we shall have results somewhat different, and more accordant at the two stations. These are contained in the following Tables, of which the first gives V = 42° nearly, the second 47° nearly, the difference being in the same direction as before.

Brientz.

First Series.			Second Series.		
Hour.	Log *v*.	*x*.	Hour.	Log. *v*.	*x*.
9	1·4065	1529	$8\frac{1}{4}$	1·3579	1952
11	1·4579	1113	10	1·4409	1244
1	1·5024	1111	12	1·4857	1073
3	1·3838	1529	2	1·4409	1242
$4\frac{1}{2}$	1·2095	2874	4	1·2989	2184
Sum.	6·9601	8156	Sum.	7·0243	7695
Mean. . . .	1·3920	1631	Mean. . . .	1·4049	1539

Taking V = 1, $[v] =$ ·7827, $m =$ ·0001402
V in actinometric degrees = 41°·75.

Faulhorn.

First Series.			Second Series.		
Hour.	Log *v*.	*x*.	Hour.	Log *v*.	*x*.
$8\frac{1}{4}$	1·3892	1501	$7\frac{1}{2}$	1·2601	2197
10	1·5250	957	9	1·4639	1176
12	1·5717	827	11	1·5866	857
2	1·5224	958	1	1·5353	857
4	1·3802	1683	3	1·4698	1178
			$4\frac{1}{2}$	1·3304	2214
Sum.	7·3885	5926	Sum.	8·6461	8479
Mean. . . .	1·4777	1185	Mean. . . .	1·4410	1413

Taking V = 1, $[v] =$ ·7544, $m =$ ·0001609
V in actinometric degrees = 46°·60.

104 Again, if we compare the *whole* observations at the Faulhorn in one series with the *whole* observations at Brientz in another, we find

	Mean value of log *v*.	Mean value of *x*.
Brientz	1·3984	1585
Faulhorn	1·4577	1310

from which we obtain,

taking V = 1, $[v] =$ ·6857,

agreeing (as might be expected) almost exactly with the result of Art. 69. p. 252, where the same quantity was deduced from the separate simultaneous observations. Also we have

$m =$ ·0002156
V in actinometric degrees = 54°·97.

SECTION VII.—*Other Observations in* 1832.

105. The following miscellaneous observations of the actinometer and other meteorological instruments were made simultaneously, or nearly so, at the Faulhorn by M. KÄMTZ, and at various inferior stations by myself in September 1832. If they afford no other immediate result, they at least show how unavailing such observations are unless made under the *most* favourable circumstances with respect to weather. I shall offer but few remarks upon these Tables, which after what has been said explain themselves sufficiently. I will add, however, that in point of care and continuity of observation, these observations are in general equally worthy of confidence with those of the 25th of September.

TABLE G.—Meteorological Observations at various Stations, September 1832.

Place.	Date.	Mean time.	Apparent time.	Barometer, French.	Attached Therm. Reaum.	Barometer in millimetres.	Attached therm. Cent.	Barometer at 0°.	Detached therm. Fahr.	Moist therm.	Diff.	Dryness. Elasticity of vapour.	Dryness. Relative Dampness.
	1832.	h m	h m	in. lin. 16ths.	°				°	°	°	inch.	
Near Grindelwald	September 16	9 6	9 14	24 4 13	9·4	660·52	11·7	659·09	43·5	40·0	3·5	·229	$\frac{229}{299}$ = ·766
Rosenlaui	September 16	12 15	12 23	24 2 10	12·6	655·61	15·7	653·69	46·5	39·4	7·1	·189	$\frac{189}{331}$ = ·571
Guttannen	September 17	12 20	12 28	25 1 4	13·2	679·57	16·5	677·57	64·2	56·0	8·2	·374	$\frac{374}{601}$ = ·622
Interlaken	September 22	10 44	10 52	26 8 14	14·2	723·85	17·7	721·49	67·0				
Interlaken	September 22	11 48	11 56	26 8 11	14·7	723·42	18·4	720·98	62·5	55·0	7·5	·360	$\frac{360}{587}$ = ·613
Grindelwald	September 23	11 15	11 23	25 2 7	10·2	682·25	12·7	680·65	59·0	52·5	6·5	·339	$\frac{339}{506}$ = ·670
Grindelwald	September 23	12 20	12 28	25 2 7	19·8	682·25	24·7	679·14	60·5	54·3	6·2	·368	$\frac{368}{532}$ = ·692
Giesbach............	September 24	3 5	3 13	26 6 0	17·0	717·35	21·2	714·53	68·0	61·5	6·5	·480	$\frac{480}{681}$ = ·705
Lungernsee	September 26	11 2	11 10	26 3 12	12·5	712·28	15·6	710·24	60·4	54·2	6·2	·365	$\frac{365}{530}$ = ·689
Near Sarnen	September 26	12 1	12 9	26 10 14	17·0	755·43	21·2	752·48	67·0	58·2	8·8	·395	$\frac{395}{659}$ = ·599

TABLE H.—Meteorological Observations on the Faulhorn, September 1832.

Place.	Date.	Mean time.	Apparent time.	Barometer, French, at 0° C.		Detached therm. Reaum.	Moist therm. Reaum.	Detached therm. Fahr.	Moist therm. Fahr.	Diff.	Elasticity of vapour.	Relative Dampness.
		h m	h m	lines.	mm.	°	°	°	°	°	inch.	
Faulhorn...	September 16	9 6	9 14	243·47	549·23	−6·0	−6·5	18·5	17·4	1·1	·109	$\frac{109}{122}$ = ·893
Faulhorn...	September 16	11 21	11 29	243·79	549·95	−4·0	−5·2	23·0	20·3	2·7	·109	$\frac{109}{144}$ = ·757
Faulhorn...	September 17	12 20	12 27	245·51	553·83	+1·3	0·0	34·9	32·0	2·9	·175	$\frac{175}{221}$ = ·792
Faulhorn...	September 22	10 37	10 45	246·88	556·91	+3·0	+1·2	38·7	34·6	4·1	·185	$\frac{185}{253}$ = ·731
Faulhorn...	September 23	11 12	11 20	247·01	557·21	+3·4	+2·2	39·6	36·9	2·7	·209	$\frac{209}{261}$ = ·801
Faulhorn...	September 24	3 4	3 12	247·20	557·64	+3·8	+2·6	40·5	37·8	2·7	·222	$\frac{222}{269}$ = ·825
Faulhorn...	September 26	11 3	11 11	246·43	555·90	+6·1	+1·4	45·7	35·1	10·6	·134	$\frac{134}{322}$ = ·416
Faulhorn...	September 26	12 3	12 11	246·34	555·70	+5·4	+0·4	44·1	32·9	11·2	·112	$\frac{112}{305}$ = ·367

Table I.—Miscellaneous Actinometer Observations, September 1832.

Place.	Date.	Hour. From	Hour. To	Mean.	Apparent time.	Each observation.	Lower Station. Actinometer, G. 7.	Mean.	Reduced to 60 sec.	Reduced to B. 2.	Faulhorn. Hour. From	Faulhorn. Hour. To	Faulhorn. Mean.	Faulhorn. Apparent time.	Faulhorn. Actinometer. B. 2. each observation 60 sec.	Mean.
	1832.	h m	h m	h m	h m	s					h m	h m		h m		
Near Grindelwald..	Sept. 16.	8 56¾	9 6	9 1½	9 9	30	13·6; 13·2; 15·2; 15·2; 15·6; 15·2; 14·0	14·6	16·3	19·0	8 46	8 56	8 51	8 59	29·4; 29·4; 29·0; 29·7	29·4
Rosenlaui............	Sept. 16.	11 49½	11 57	11 53¼	12 1	30	18·2; 22·3; 17·0; 19·4; 17·8	18·9	21·1	24·6	11 49½	11 57	11 53¼	12 1	39·8; 40·7; 38·9	39·8
Rosenlaui............	Sept. 16.	11 56½	12 3½	12 0	12 8	30	20·3; 21·1; 20·4; 20·1; 18·5	20·1	22·4	26·2						
Rosenlaui............	Sept. 16.	12 3	12 9½	12 6¼	12 14	30	18·4; 19·1; 19·2; 18·9; 19·7	19·1	21·3	24·9						
Guttannen..........	Sept. 17.	11 54	11 59½	11 56¾	12 5	30	18·9; 25·1; 28·9; 32·8	26·4	29·4	34·3	11 46	11 55½	11 50¾	11 59	48·0; 44·6; 43·6; 44·4	45·2
Guttannen..........	Sept. 17.	12 6½	12 15	12 10¾	12 19	30	25·3; 26·9; 27·5; 29·7; 27·1; 27·8; 27·8	27·4	30·5	35·6	12 10	12 19½	12 14¾	12 23	44·7; 43·9; 42·5; 43·0	43·5
Interlaken..........	Sept. 22.	10 44½	10 52½	10 48½	10 57	30	18·9; 19·3; 19·7; 21·1; 20·8; 20·8	20·1	22·4	26·2						
Interlaken..........	Sept. 22.	11 48½	11 55½	11 52	12 0	30	21·4; 22·7; 21·5; 21·6; 22·6; 24·0	22·3	24·8	29·0						
Interlaken..........	Sept. 22.	11 55	12 2½	11 58¾	12 7	30	20·1; 19·9, 20·0; 22·9; 24·0; 21·0	21·3	23·7	27·7						
Interlaken..........	Sept. 22.	12 2	12 9½	12 5¾	12 14	30	23·2; 23·9; 24·1; 27·1; 27·2; 26·4	25·3	28·2	32·9						
Grindelwald..	Sept. 23.	10 22½	10 28½	10 25½	10 34	30	17·0; 19·4; 17·0; 18·9; 17·2	17·9	19·9	23·2	10 27½	10 37	10 32¼	10 40	36·4; 36·9; 37·3; 36·5	36·8
Grindelwald.........	Sept. 23.	10 28	10 35	10 31½	10 40	30	17·3; 18·2; 19·0; 19·9; 19·8	18·8	20·9	24·4	....	....	....	....	33·8; 34·0; 35·2; 36·7	34·9
Grindelwald.........	Sept. 23.	10 53	11 8	11 0½	11 9	60	21·9; 22·5; 22·8; 23·5; 23·7; 24·1	23·1	....	27·0						
Grindelwald.........	Sept. 23.	11 7½	11 14½	11 11	11 19	30	22·7; 23·0; 21·7; 24·0; 21·3; 22·0	22·4	25·0	29·2	11 12	11 21	11 16½	11 25	35·0; 34·3; 35·5; 36·0	35·2
Grindelwald.........	Sept. 23.	11 40	11 45	11 42½	11 51	30	22·0; 22·7; 22·9; 23·9	22·9	25·5	29·8 } 31·2	11 39	11 48½	11 43¾	11 52	36·5; 35·1; 36·1; 37·8	36·4
Grindelwald.........	Sept. 23.	11 44½	11 50	11 47¼	11 55	30	24·8; 24·9; 25·6; 24·9	25·0	27·8	32·5 }						
Grindelwald.........	Sept. 23.	11 49½	12 5	11 52¼	12 0	60	25·4; 27·6; 27·0; 26·2; 25·8; 26·9	26·5	....	30·9						
Grindelwald.........	Sept. 23.	12 6½	12 13	12 10	12 18	30	26·5; 27·2; 25·5; 25·9; 25·3; 23·6; 25·0	25·6	28·5	33·3						
Grindelwald.........	Sept. 23.	12 12½	12 19½	12 16	12 24	30	25·2; 25·5; 23·6; 23·2; 23·9; 23·7	24·2	27·0	31·5						
Giesbach............	Sept. 24.	3 5	3 10½	3 7¾	3 16	30	17·9; 15·8; 15·5; 16·6; 15·5	16·3	18·2	21·3	3 4	3 9	3 6½	3 15	31·1; 32·2	31·7
Giesbach............	Sept. 24.	3 10	3 25½	3 17¾	3 26	60	16·9; 18·3; 18·8; 16·6; 18·0; 19·0; 19·2	18·1	....	21·1						
Lungernsee..........	Sept. 26.	11 2	11 20	11 11	11 19	60	22·7; 23·1; 23·9; 24·2; 24·9; 25·2; 24·7	24·1	....	28·1	11 3¼	11 29½	11 16¼	11 24	34·0; 34·3; 33·7; 35·2 34·5; 35·1; 33·7; 36·9 34·8; 36·0; 35·6; 35·2	34·9
Near Sarnen........	Sept. 26.	12 1	12 12½	12 6¾	12 15	60	27·7; 26·2; 26·5; 27·7; 27·3	27·1	....	31·6 } 32·3	12 3	12 32¼	12 17½	12 26	40·6; 41·1; 40·9; 40·4; 41·2 40·9; 41·2; 41·4; 40·9 41·7; 40·8; 42·4; 42·2; 42·2	41·3
Near Sarnen........	Sept. 26.	12 11½	12 21	12 16¼	12 24	60	28·7; 28·0; 28·1; 28·5	28·2	....	32·9 }						

TABLE K.—Miscellaneous Comparative Observations, September 1832.

No.	Date.	Hour*.	Position of Lower Station.	Lower Station. Barometer at 0° C.	Lower Station. Det. therm. FAHR.	Lower Station. Ratio to saturation.	Lower Station. Actinometer B 2.	Faulhorn. Barometer at 0° C.	Faulhorn. Det. therm. FAHR.	Faulhorn. Ratio to saturation.	Faulhorn. Actinometer B 2.	Diff. of Pressure.	Mean Temp.	Mean Dampness.	Loss of solar intensity.	Ratio of loss to effect at upper station.	Sun's altitude.	Barom. × Sec. Z. D. Lower Station.	Barom. × Sec. Z. D. Upper Station.	Barom. × Sec. Z. D. Diff.	Diff. × Mean Dampness.
	1832.	h m		mm.				mm.				m. m.									
1.	Sept. 16.	9 4	Near Grindelwald	659·09	43·5	·766	19·0	549·23	18·5	·893	29·4	109·86	31·0	·830	10°·4	·354	31° 54′	1247·2	1039·3	207·9	172·6
				at 9h 14m																	
2.	Sept. 16.	12 1	Rosenlaui	653·69	46·5	·571	24·6	549·95	23·0	·757	39·8	103·74	34·7	·664	10·1	·254	45 54	910·3	765·8	144·5	96·0
				at 12h 23m	....	....	....	at 11h 29m													
3.	Sept. 17.	12 2	Guttannen	677·57	64·2	·622	34·3	553·83	34·9	·792	45·2	123·74	49·5	·707	10·9	·241	45 31	949·7	776·3	173·4	122·6
4.		12 21	....	at 12h 28m	....	....	35·6	....	....	....	43·5	....	....	....	7·9	·182	45 14				
5.	Sept. 22.	10 52	Interlaken	721·49	67·0																
				at 10h 52m																	
6.	Sept. 22.	11 56	Interlaken	720·98	62·5	·613															
7.	Sept. 23.	10 37	Grindelwald....	at 11h 56m	....	·655?	23·2	556·91	....	·731	36·8	124·76?	....	....	13·6	·369	39 42	......	871·9		
8.	Sept. 23.	11 22	Grindelwald....	680·65	59·0	·670	29·2	557·21	39·6	·801	35·2	123·44	49·3	·735	6·0	·170	42 24	1009·4	826·4	183·0	134·5
9.	Sept. 23.	11 52	....	at 11h 23m	....	·680	31·2	at 11h 20m	....	....	36·4	......	...	....	5·2	·143	43 8				
10.	Sept. 23.	12 28	Grindelwald....	679·14	60·5	·692															
				at 12h 28m																	
11.	Sept. 24.	3 15	Giesbach	714·53	68·0	·705	21·3	557·64	40·5	·825	31·7	156·89	54·2	·765	10·4	·328	26 35	1596·7	1246·1	350·6	268·2
				at 3h 13m																	
12.	Sept. 26.	11 21	Lungern See ..	710·24	60·4	·689	28·1	555·90	45·7	·416	34·9	154·34	53·0	·558	6·8	·195	41 15	1075·2	841·6	233·6	130·3
				at 11h 10m																	
13.	Sept. 26.	12 23	Near Sarnen ..	752·48	67·0	·599	32·3	555·70	44·1	·367	41·3	196·78	55·5	·483	9·0	·218	41 46	1129·7	834·3	295·4	142·7
				at 12h 29m	....	....	....	at 12h 11m													

* The hour here is the mean apparent time of the actinometer observations at both stations.

Remarks at the *Lower* Station.

No.
1. Hasty observations. Very clear.
2. Sky quite clear, except some passing well-defined clouds, which were avoided.
3. Very fine sky.
5. Sky quite cloudless, but not a fine blue. Observations at the window of the inn.
7. Magnificent sky. Observations at the window of the inn.
11. Quite clear. Not a cloud all day.
12. Splendid sky. A little wind.

Remarks at the *Upper* Station by M. KÄMTZ.

No.
1. Fog in the valley of Grindelwald from $9\frac{1}{2}$ to 10^h.
2. Fogs some time passing. Observations made in the intervals when none near the sun.
3. This observation and the preceding ones were made in a room. The following ones were out of doors.
8. Clouds on the Schreckhorn and northern plain.
9. After this clouds sometimes pass through the zenith and before the sun.
11. Some fogs.

Curves XXI. and XXII. Plate XXV. show the relative march of the actinometer at Grindelwald and the Faulhorn on the 23rd of September, during the latter part of which clouds appeared at the upper station which were not visible, or at least not observed, at the lower; and the effect on the inflection of the diurnal curve is the same as that noticed Art. 79, p. 255.

106. Although these observations were never made except when the sun appeared to shine through a clear blue sky, the rate of extinction is enormously greater than in the formerly described more favourable circumstances. By selecting the observations directly comparable, and reducing them as in Art. 62, I have found an absorption equal to *three-fourths* of the incident heat, the mean ratio to saturation being ·6717. But it must be confessed that no evident relation to the hygrometric condition of the air appears in the individual observations.

Section VIII.—*Observations in* 1841.

107. These were made under very far from favourable circumstances at one station only, namely, on the lower glacier of the Aar, at an elevation of about 7000 feet above the sea. Although the sky was to appearance generally clear and of a deep blue during the continuance of these observations, the occasional formation of slight clouds, and the feeble degree of dryness, considering the elevation, explains the comparatively great opacity of the atmosphere which we deduce from these observations. The instrument was a different one and partly on a different construction from those formerly used, and they have not been compared, consequently the actinometric degrees are not convertible into one another. It may be doubted whether the surface of a glacier is not a very bad position for such observations, owing to the stratum of moist air, which in summer must generally rest upon it during the heat of the day, and the glare from the adjoining mountains is an evil not wholly to be avoided. I must state, however, that numerous comparative experiments which I made on this occasion with the actinometer and with Leslie's photometer, convinced me of the remarkable constancy and truth of the indications of the former. My immediate object was to verify a conjecture which I had published* respecting an anomaly in solar radiation described by Dr. Richardson in the arctic regions, as measured by the statical thermometric effect. The anomaly was that the maximum occurred in April or May, instead of in June or July, as elsewhere; and the explanation I gave was that the disappearance of the snow from the earth's surface in the month of May diminished the solar effect more than the sun's greater elevation increased it. This was confirmed by finding the *enormous* indications given by Leslie's photometer on the glacier of the Aar, at a time when the sun's rays were not peculiarly intense, which I ascribed to the glittering reflection from the surface of the glacier, and from the amphitheatre of snowy mountains. When the instrument was placed on a rock, or merely on a piece of black wax-cloth laid upon the snow, it sunk in a very remarkable manner. The actinometer, on the other hand, when supported on a small box, so as just to avoid contact with the snow, gave appreciably the same result in both situations.

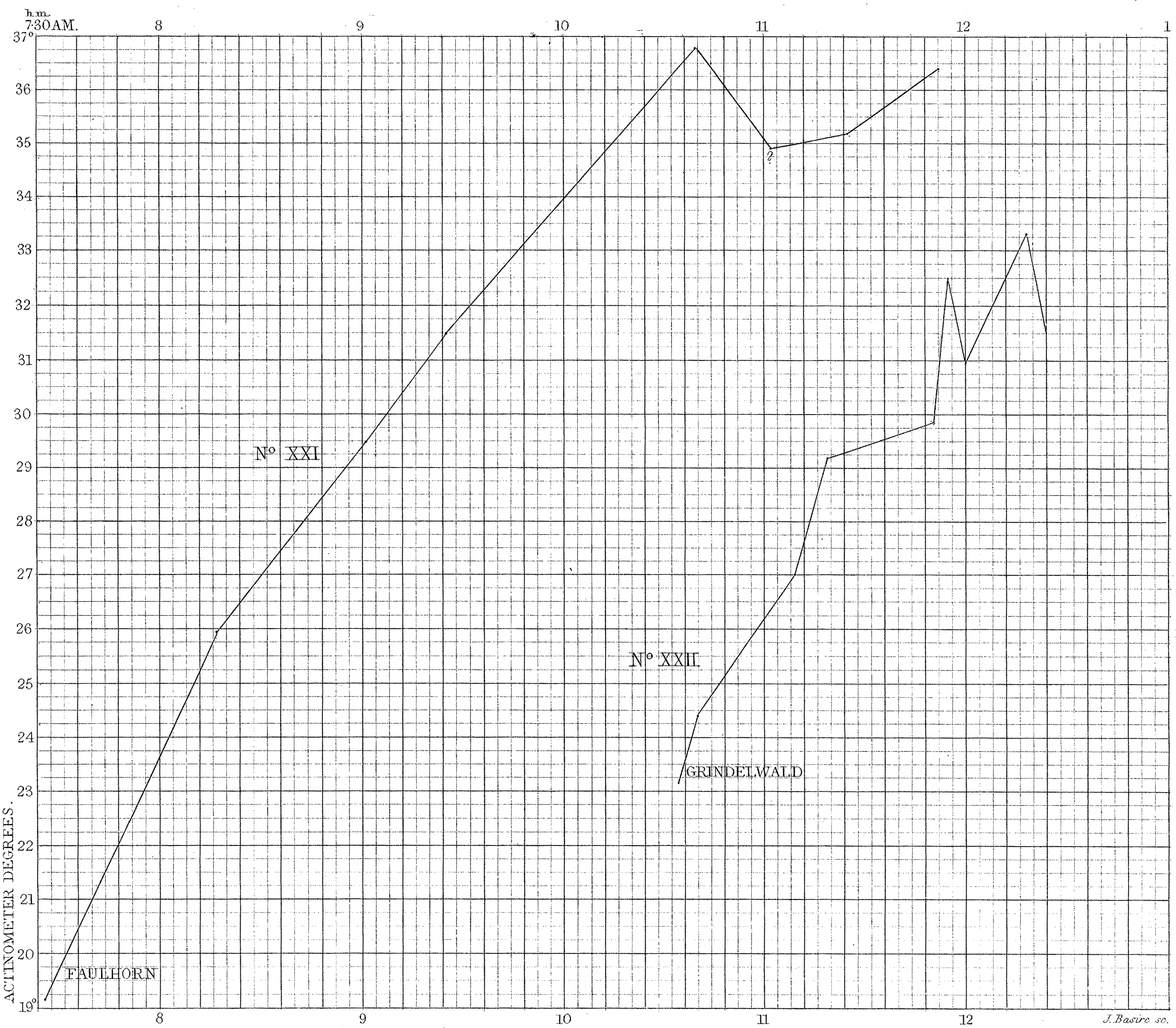
108. The following are the most available meteorological observations made in 1841:

* Jameson's Edinburgh New Philosophical Journal for 1841.

COMPARATIVE INTENSITY OF SOLAR RADIATION.

GRINDELWALD & FAULHORN — SEPT^{R} 23RD 1832.

Phil. Trans. MDCCCXLII. *Plate XXV. p. 268.*



J. Basire sc.

TABLE L.—Observations on the Lower Aar Glacier, 13th and 14th August 1841.

Mean height on the Barometer 568^{mm}. Latitude 46° 33′ N.

	Hour.			Thermometer.				Diff. Fahr.	Elasticity of vapour.	Relative dampness.	Actinometer S. 1.	Mean.	Sun's altitude.	Thickness of atmosphere = Bar. × sec. Z. D.	Remarks.
	From	To	Mean.	Dry. Reaum.	Moist. Reaum.	Dry. Fahr.	Moist. Fahr.				Each observation 60 seconds.				
1841. Aug. 13.	h m 6 41	h m 6 52	h m 6 46	....	....	....	....	....	....	....	6·2; 8·0; 9·3; 10·1; 10·6	8·84	° ′ 19 29	mm. 1703·0	Aug. 13. Light clouds generally visible in some part of the sky, but not so as to conceal the sun at any time.
	7 57	8 11½	8 4	....	....	....	....	....	....	....	10·2; 12·5; 14·8; 17·0; 15·5; 14·9	14·15	31 51	1076·4	
	9 0	9 11½	9 6	1·8	− 1·2	36·0	29·3	6·7	·124	$\frac{124}{230}$ = ·539	18·4; 15·8; 16·4; 20·0; 21·8	18·5	41 55	850·2	
	10 0	10 13½	10 7	4·9	+ 2·0	43·0	36·5	6·5	·178	$\frac{178}{293}$ = ·607	24·7; 20·8; 21·3; 21·5; 20·6; 23·1	22·0	50 28	736·5	
	12 0	12 16	12 8	5·9 at 12^h 30^m	+ 2·6	45·2	37·8	7·4	·182	$\frac{182}{317}$ = ·574	25·9; 27·2; 26·9; 27·8; 28·4; 26·0	27·03	58 8	668·8	
	2 44	2 59½	2 52	5·4	+ 3·1	44·1	38·9	5·2	·209	$\frac{209}{305}$ = ·685	20·7; 20·5; 22·0; 20·7; 29·0	22·58	41 53	850·8	
	4 0	4 9½	4 5	5·0	+ 2·5	43·2	37·6	5·6	·195	$\frac{195}{296}$ = ·659	21·6; 23·1; 23·1; 20·3	22·02	30 15	1127·5	
	5 7	5 17¼	5 12	....	....	....	....	....	....	....	20·0; 19·3; 20·5; 20·1	20·0	18 45	1767·0	
Aug. 14.	6 18	6 27	6 22½	1·8	− 0·2	36·0	31·6	4·4	·159	$\frac{159}{230}$ = ·691	8·4; 8·55; 9·3; 10·5	9·19	15 52	2077·5	
	8 51½	9 10¼	9 1	7·7 at 10^h 30^m	+ 3·0	49·3	38·7	10·6	·162	$\frac{162}{365}$ = ·444	15·7; 16·95; 20·25; 21·6; 23·3; 19·2; 22·95; 22·55	20·3	40 51	868·4	
	11 55	12 13	12 4	6·7	+ 3·0	47·1	38·7	8·4	·181	$\frac{181}{338}$ = ·536	21·1; 23·9; 23·2; 23·3; 23·8; 25·1	23·4	57 46	671·5	
	12 22¼	12 31¼	12 27	6·6	+ 3·0	46·8	38·7	8·1	·184	$\frac{184}{335}$ = ·549	25·4; 26·3; 26·7; 26·5	26·22	57 18	675·0	A few vapours, very light, appear. Wind rises.

109. If we compare the longest with the shortest columns on the 13th of August, we obtain in the notation of Art. 100., taking V = 1, $[v] = \cdot 6403$, $m = \cdot 0002549$.

V in degrees of actinometer S. 1. = 35°·22.

And on the 14th of August, $[v] = \cdot 5680$, $m = \cdot 0003232$.

V in degrees of actinometer S. 1. = 40°·87.

110. These results are on the logarithmic hypothesis. If, however, we project the observations, as we have done those of the 25th of September, 1832, we find that they cannot be represented by a simple geometrical progression. An interpolating curve, which will satisfy them sufficiently well, is a logarithmic curve, whose asymptote is distant 7°·25 actinometric degrees from the axis of x. The curve is represented in Plate XXVI. Curve XXIII. The constants of the curve were derived by a graphical process, as in the former case, tangents having been first drawn to the empirical curve, whence the following velocities of extinction were deduced, which are compared with a formula of the same form as in Art. (93.), namely

$$-\frac{dv}{dx} = av - b.$$

The values of a and b were deduced from the projection, Curve XXIV. in the same Plate, which gives 7°·25 for the intensity when the rate of extinction is zero; and

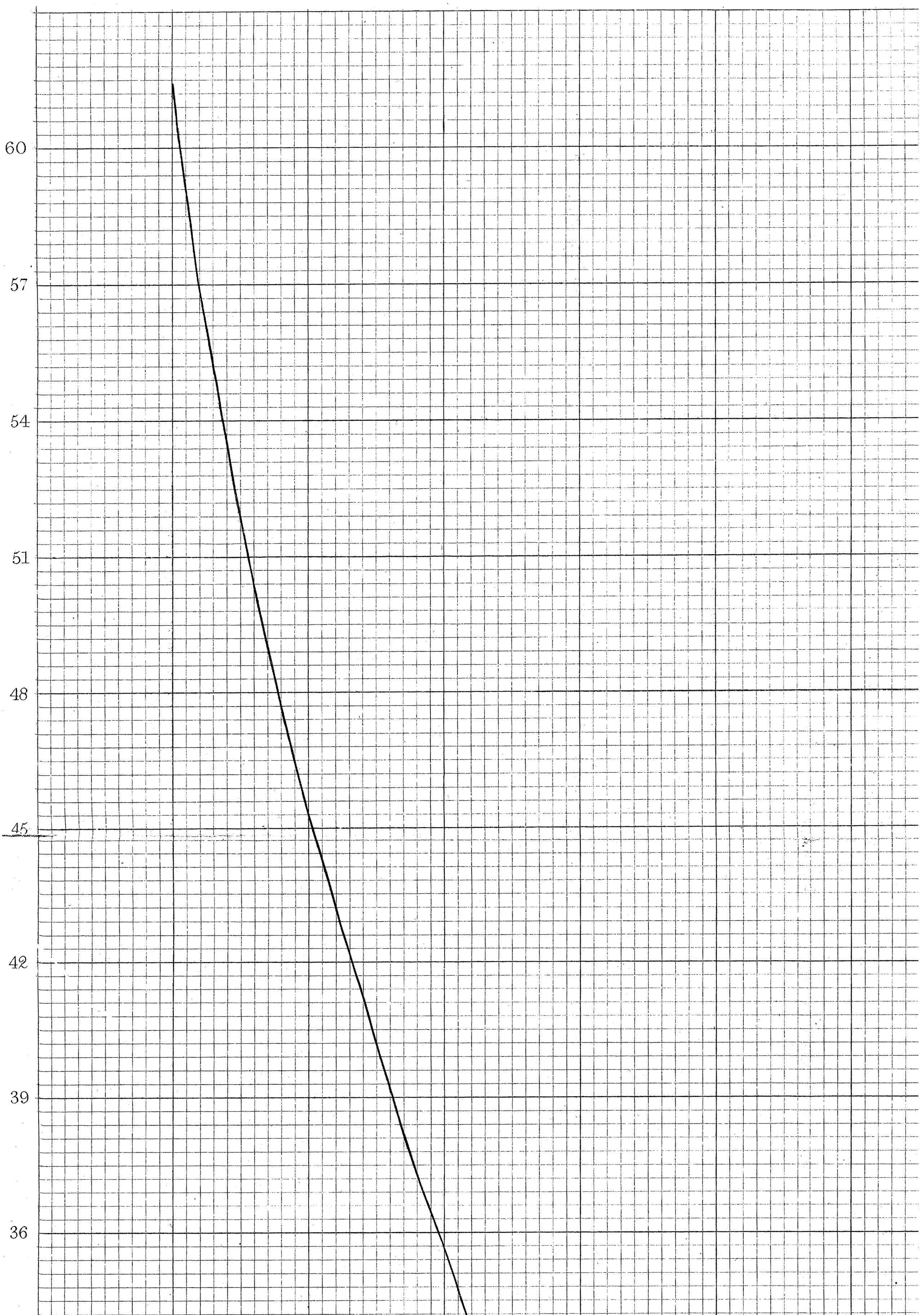
$$\left.\begin{aligned} a &= \cdot 00192 \\ b &= \cdot 01396 \end{aligned}\right\} \text{for } 1^{\text{mm}} \text{ of thickness.}$$

Intensity.	Rate of loss for 500 millimetres. Observed.	Calculation.	Difference.
25	16·5	17·04	+0·54
20	12·6	12·24	−0·36
15	8·0	7·44	−0·56
10	2·3	2·64	+0·34

Whence the following points of the curve have been computed from forms similar to those of Art. 96, $\frac{b}{a}$ being here 7·25.

Thickness, or x in millimetres of mercury.	Intensity, or degrees of Actinometer S. I.
0	70·74 + 7·25 = 77·99
500	25·51 + 7·25 = 32·76
1000	9·20 + 7·25 = 16·45
1500	3·32 + 7·25 = 10·57
2000	1·20 + 7·25 = 8·45
2500	0·43 + 7·25 = 7·68

111. Hence it appears that a general analogy holds with results of the 25th of September 1832. It is even not improbable that the degrees of the two instruments do not greatly differ in value, and that the lower indications in 1841 are due solely to the greater opacity of the air, marked by the rapid decline of the curve, and (as might be anticipated) the lower value of the limiting or final intensity.



60
57
54
51
48
45
42
39
36

Nº XXIII

INTENSITY OF SOLAR RADIATION,

BY ACTINOMETER S. 1. ON THE 13TH & 14TH AUGUST 1841.

AT THE LOWER AAR GLACIER.

The dotted lines refer to the 13th August;
The full lines to the 14th.

LOWER A

Hour Appt. Time	13th Aug. Thickness Atmosphere *Millimetres*	Actinometer
6. 46	1703·0	8·84
8. 4	1076·4	14·15
9. 6	850·2	18·5
10. 7	736·5	22·0
12. 8	668·8	27·03
2. 52	850·8	22·58
4. 5	1127·5	22·02
5. 12	1767·0	20·0

BY

Intensity	Rate o for 50 *Obse*
25	16
20	12
15	8
10	2

N° XXIV

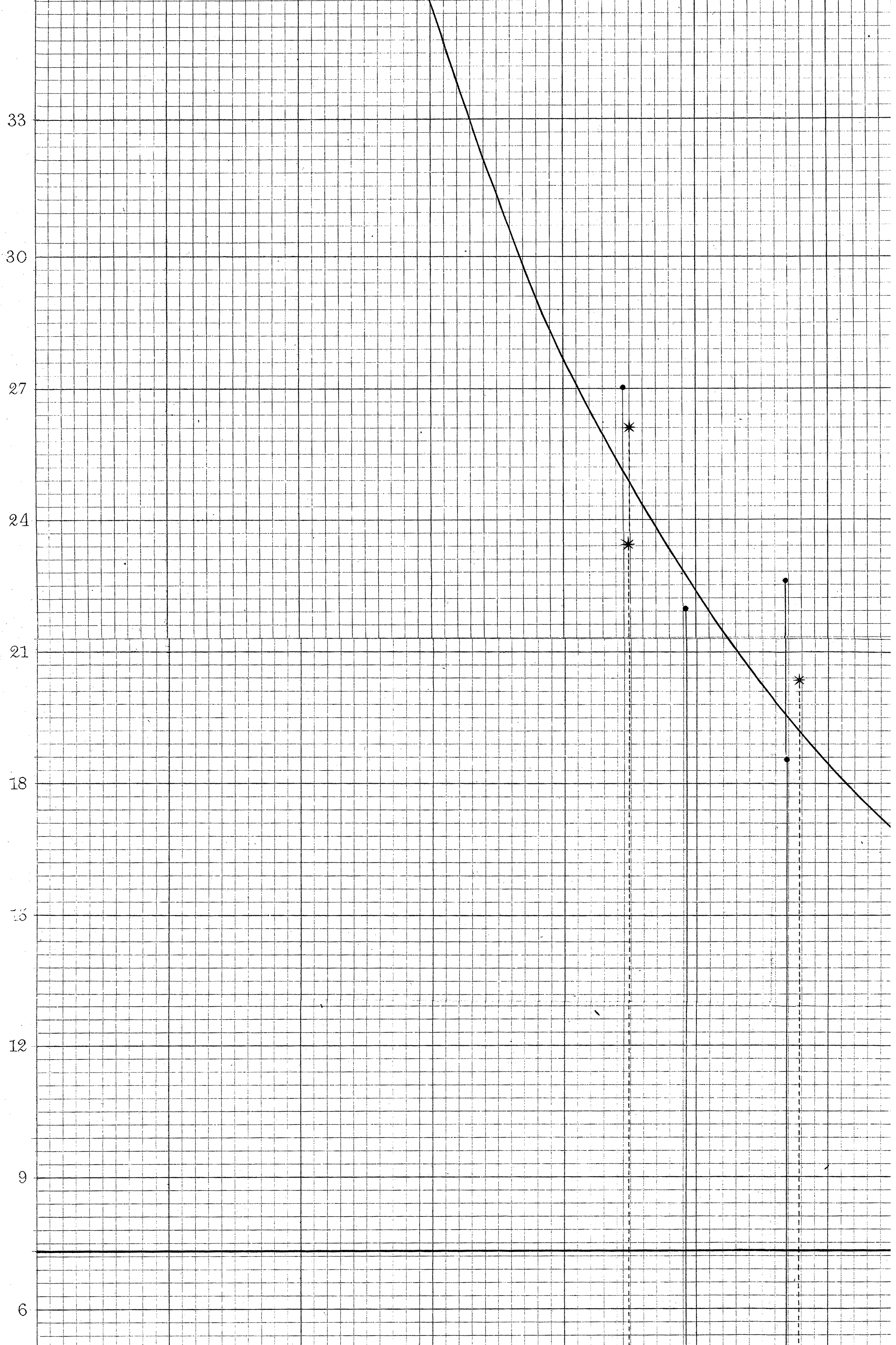
ER AAR GLACIER 1841

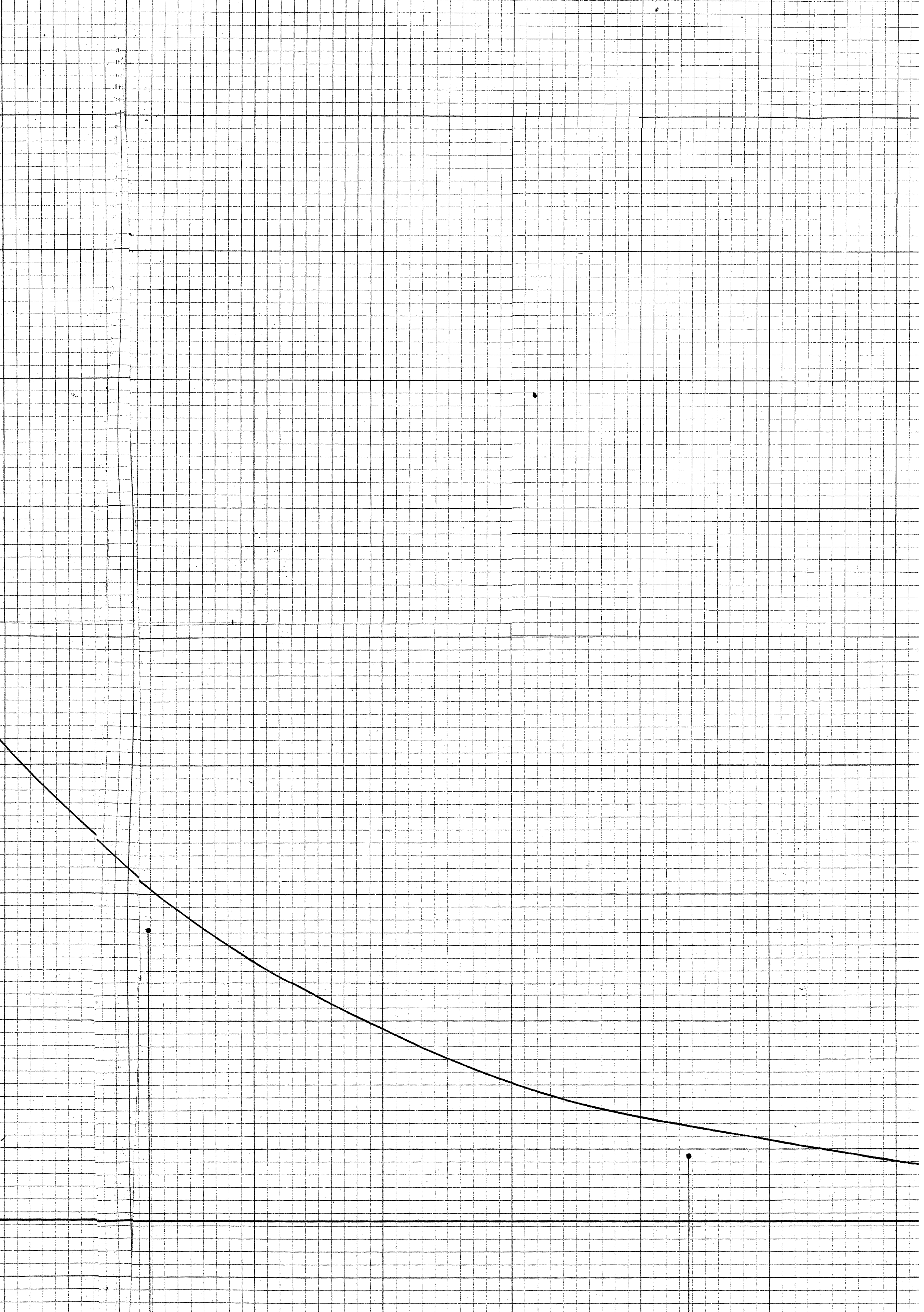
inometer	Hour Appt. Time	14th Aug. Thickness Atmosphere *Millimetres*	Actinometer
8·84	*6. 22½*	2077·5	9·19
14·15			
18·5	*9. 1*	868·4	20·3
22·0			
27·03	*12. 4*	671·4	23·4
22·58	*12. 27*	675·0	26·22
22·02			
20·0			

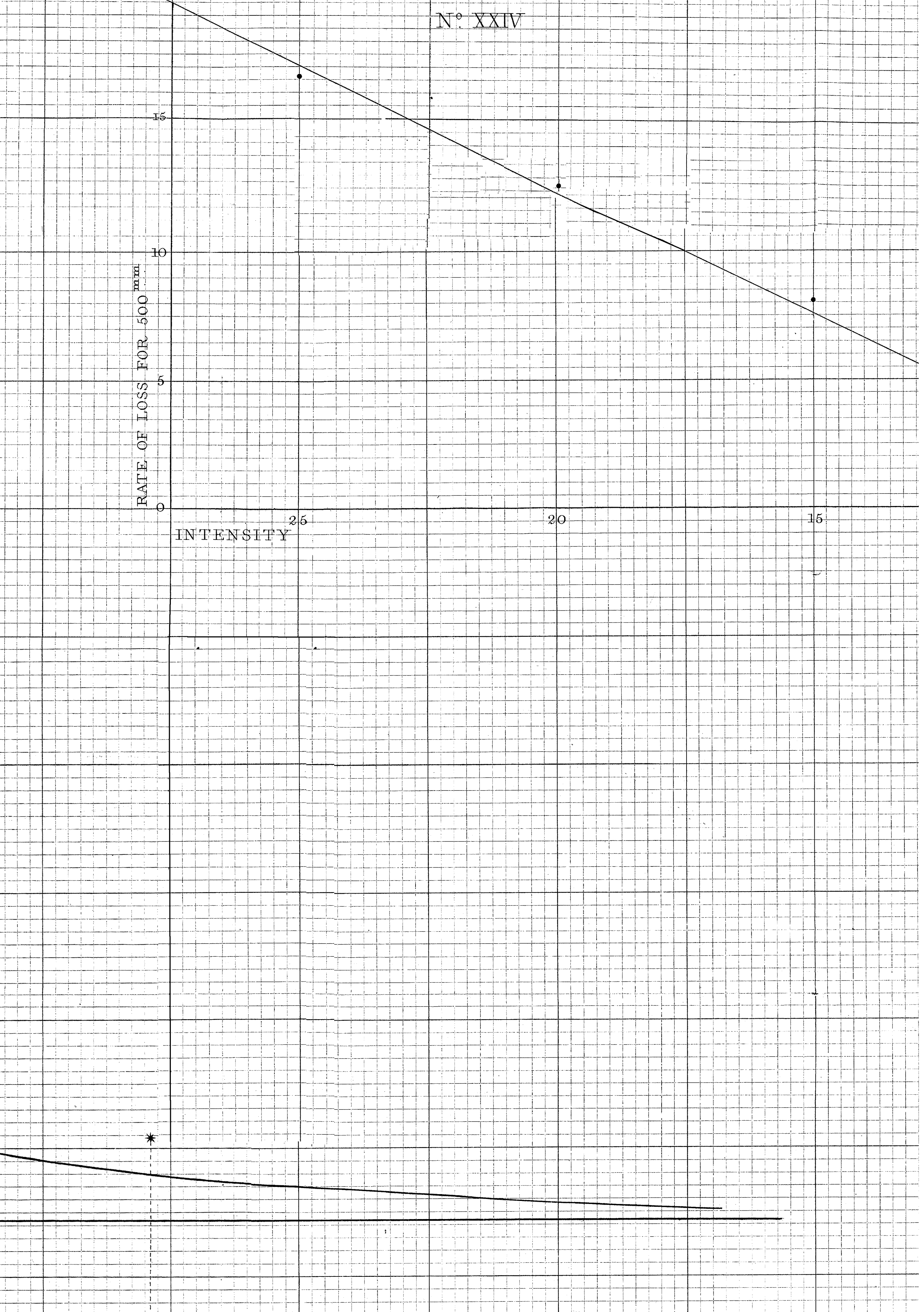
BY CURVE XXIV.

Rate of Loss for 500mm.

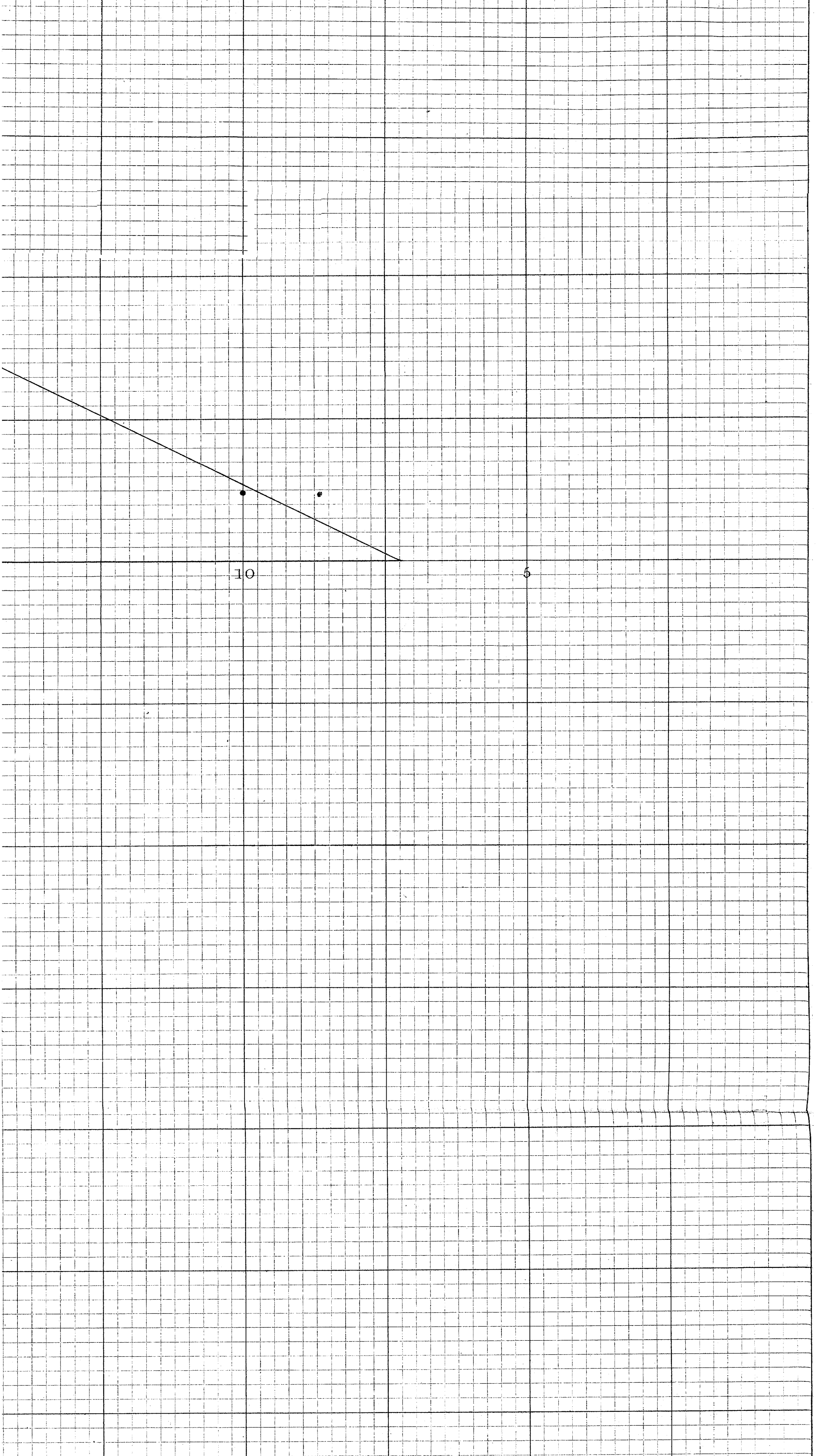
Observed	*Calculated*	*Differences*
16.5	17.04	.54
12.6	12.24	-.36
8.0	7.44	-.56
2.3	2.64	.34

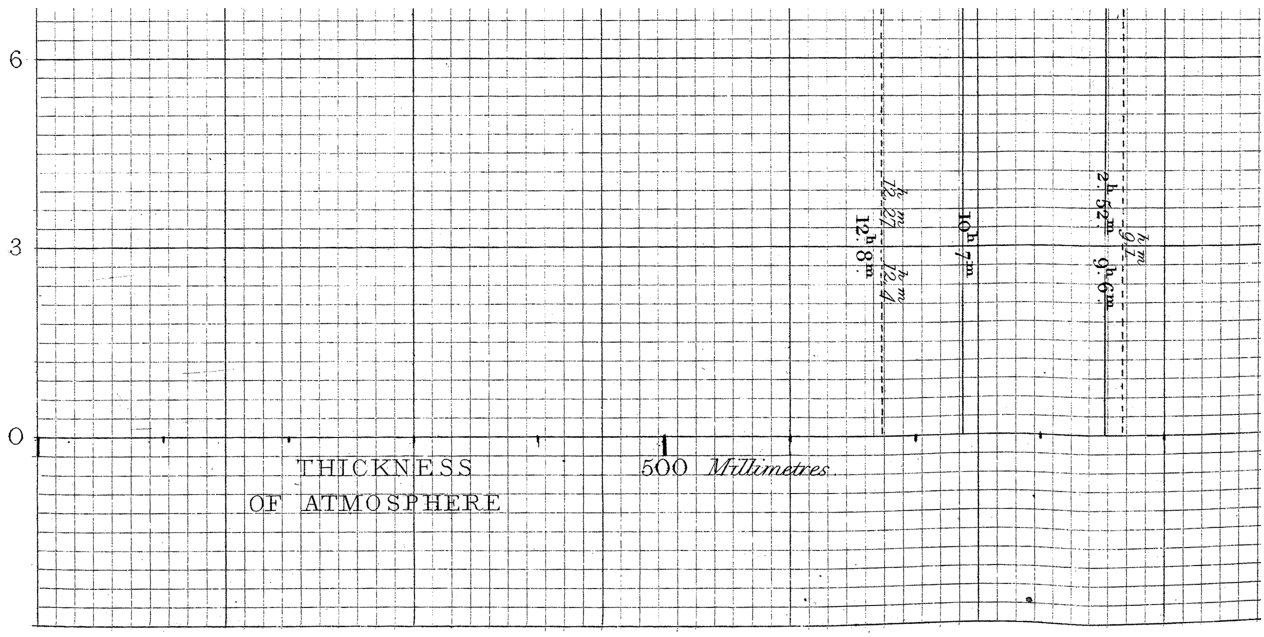

33
30
27
24
21
18
15
12
9
6

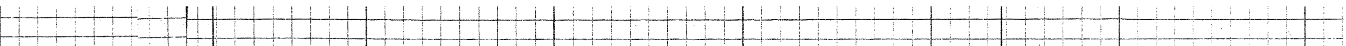


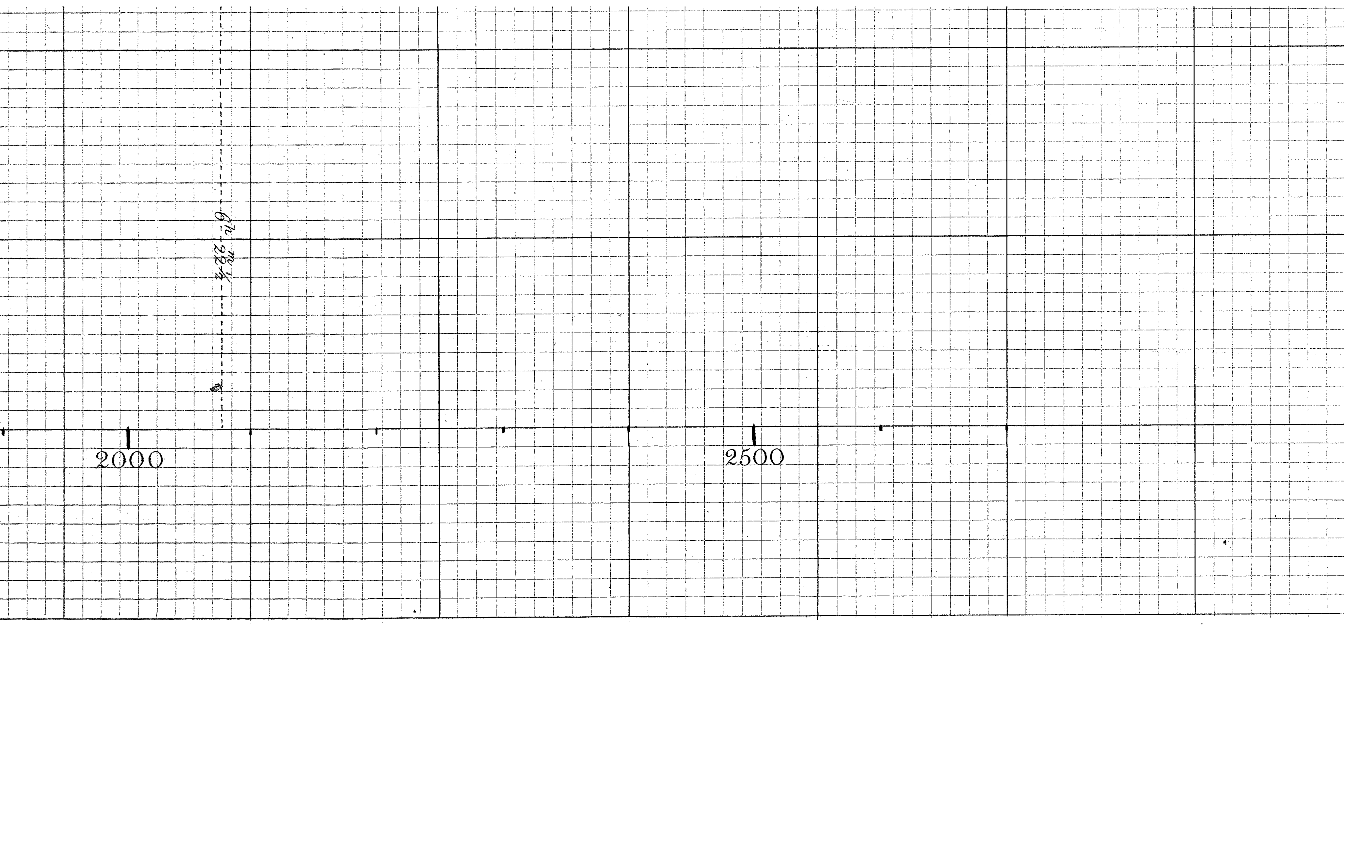


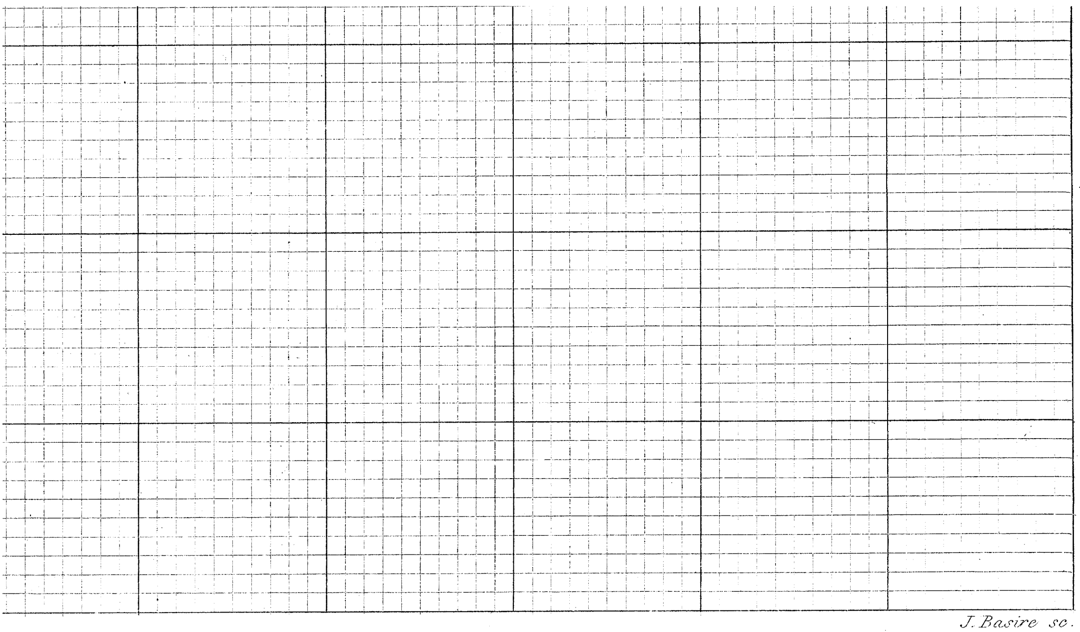
N° XXIV
RATE OF LOSS FOR 500 mm
15
10
5
0
INTENSITY
25
20
15


10
5


6
3
0
THICKNESS
OF ATMOSPHERE
500 Millimetres
12h 8m
10h 7m
2h 52m
9h 6m


1000
1500
8h 4m
6h 46m


6h 22½m
2000
2500



J. Basire sc.

Section IX.—*Conclusions.*

112. On the whole, it appears from the facts and reasonings of this paper,—

1. That the absorption of the solar rays by the strata of air to which we have immediate access, is considerable in amount for even moderate thicknesses.

2. That the diurnal curve of solar intensity has, even in its most normal state, several inflections, and that its character depends materially upon the elevation of the point of observation.

3. That the approximations to the value of extra atmospheric solar radiation, on the hypothesis of a geometrical diminution of intensity, are inaccurate.

113. The coincidence found by M. Pouillet (Art. 22.) between the logarithms of the intensities and the thicknesses, may be ascribed to his having used a formula which gives the *greater* thicknesses sensibly too small, and thus makes an accidental compensation. Perhaps another accidental compensation may be found in the continuity of the Curve XV. I have mentioned (Art. 82.) that I expected to find a different law for extinction in the higher and lower regions of the atmosphere. It may be that the greater purity of the air in the higher regions, and its great dryness, counterbalance the greater absorptive power which we have attributed (Art. 85.) to the first portions of an absorbing medium traversed by light or heat.

114. We further conclude,—

4. That the tendency to absorption through increasing thicknesses of air is a diminishing one. That in fact the absorption almost certainly reaches a limit, beyond which no further loss will take place by an increased thickness of similar atmospheric ingredients. That the residual heat (tested by the absorption into a blue liquor) may amount to from a half to a third of that which reaches the surface of the globe after a vertical transmission through a clear atmosphere.

5. That the law of absorption in a clear and dry atmosphere, equivalent to between one and four times the mass of air traversed vertically, may be represented (within those limits) by an intensity diminishing in a geometrical progression, *plus* a constant quantity, which is the limiting value already mentioned. Hence the amount of vertical transmission has always hitherto been greatly overrated, or the value of extra atmospheric solar radiation greatly underrated.

6. The value of extra atmospheric solar radiation upon the hypothesis of the above law being generally true, is 73° of the actinometer marked B. 2. The limiting value of the solar radiation, after passing through an *indefinite* atmospheric thickness, is 15°·2.

7. The absorption in passing through a vertical atmosphere of 760 millimetres of mercury, is such as to reduce the incident heat from 1 to ·534.

8. The physical cause of this law of absorption appears to be the non-homogeneity of the incident rays of heat; which by parting with their more absorbable elements become continually more persistent in their character, as Lambert and others have shown to take place where plates of glass are interposed between a source of heat and a thermometer.

9. Treating the observations on BOUGUER's hypothesis of an *uniform ratio* of extinction to the intensity of the incident ray, we obtain for the value of the vertically transmitted share of solar heat in the entire atmosphere:—

By the *relative* intensities at Brientz and the Faulhorn, Art. 69. . .	·6842
By the observations at the Faulhorn alone, 1st method, Art. 101. .	·6848
By the observations at the Faulhorn alone, 2nd method, Art. 103. .	·7544
By the observations at Brientz alone, 1st method, Art. 101. . . .	·7602
By the observations at Brientz alone, 2nd method, Art. 103. . .	·7827

ADDITIONAL NOTES.

NOTE A.—*On the Absolute Values of the Degrees of the Actinometers employed.*

Since writing this paper Sir JOHN HERSCHEL, to whom I submitted the results, has favoured me with the following information:—

"It happens very fortunately that as regards actinometer G, No. 7, I find a series of direct comparisons of this with my standard H, No. 8, which is that I employed to determine the parts of its scale in *actines*" [see Art. 19. of this paper]. "The series in question was made March 15, 1828, and gave the results of alternate sets as follows:—

G. 7.	H. 8.
20°·8	21·3
21·5	21·6
20·7	21·2
20·75	21·0
20·65	19·9
20·9	21·0
	Rejecting the last 21·3

Whence it results that the same radiation which raises G. by 209 parts would raise H. by 213 parts; or 1 part of G. 7. is equivalent to $\frac{213}{209} = 1·019$ parts of H. 8.

"Now the value of 1° of the scale of H. 8. I ascertained by an elaborate series of experiments under a sun as nearly vertical as the Cape latitude would allow, and in eminently favourable circumstances, to be 6·093 actines; so that one part of G. 7. is equivalent to 6·209 actines."

In the preceding paper the indications of G. 7. have been reduced to those of B. 2. Assumed as a standard, it has been found (Art. 54.) that the factor of reduction for the readings of G. 7. to those of B. 2. is 1·168. Hence to invert the process or reduce B. 2. to G. 7. we must multiply by

$$\frac{1}{1·168} = 0·856;$$

and to reduce the readings of B. 2. into "actines" we have the factor $0{\cdot}856 \times 6{\cdot}209 = 5{\cdot}315$.

The scale of "actines" has been added to the margin of the Curve XV. Plate XXII.

Intensity of extra-atmospheric radiation,

$73^{\circ}{\cdot}06$ B. 2. (Article 97.) = 388·4 actines.

After vertical transmission through the atmosphere,

$39^{\circ}{\cdot}03$ B. 2. = 207·4 actines.

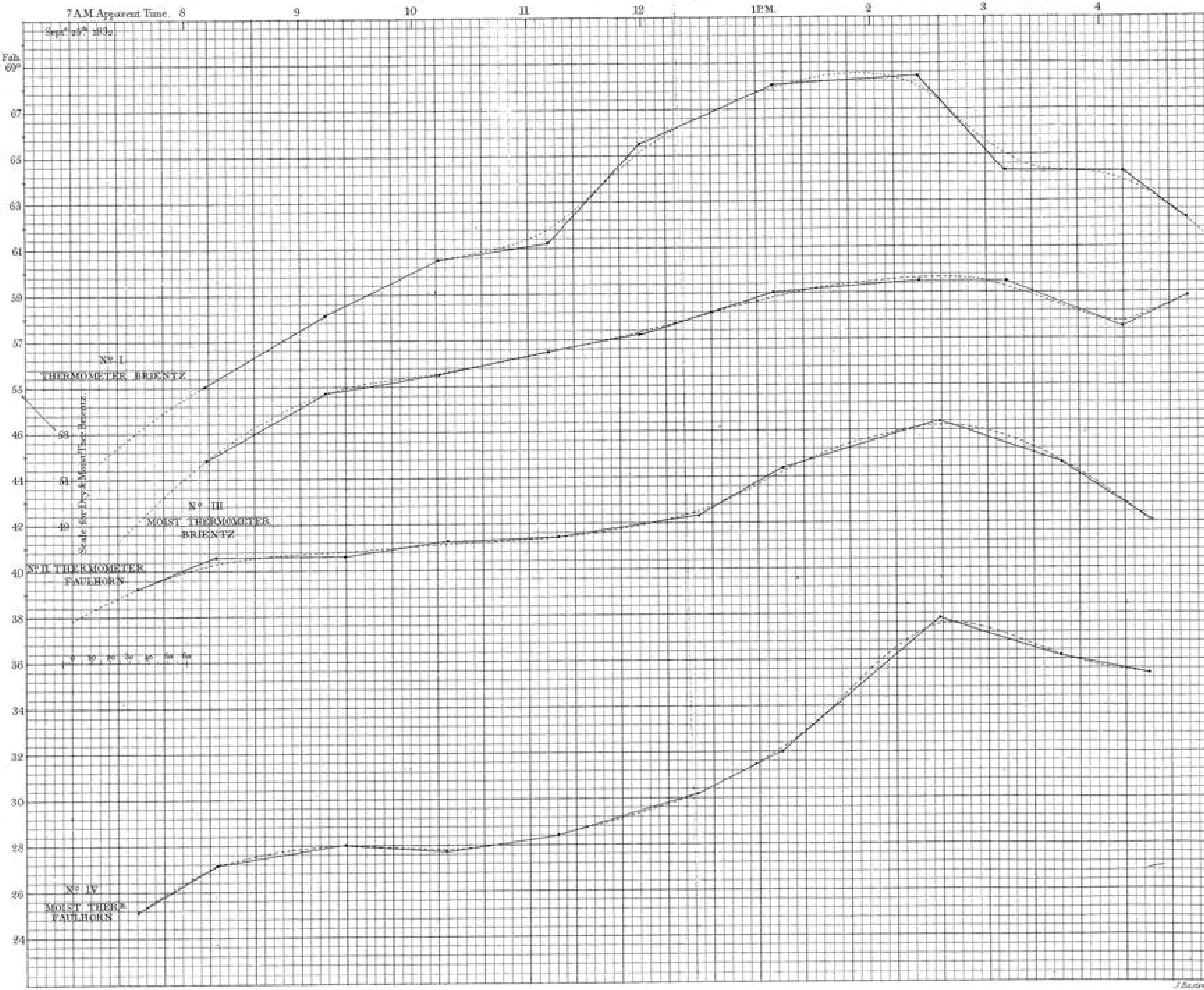
Residual intensity after an indefinite transmission,

15·2 B. 2. = 80·8 actines.

Note B.—*On Article* 48.

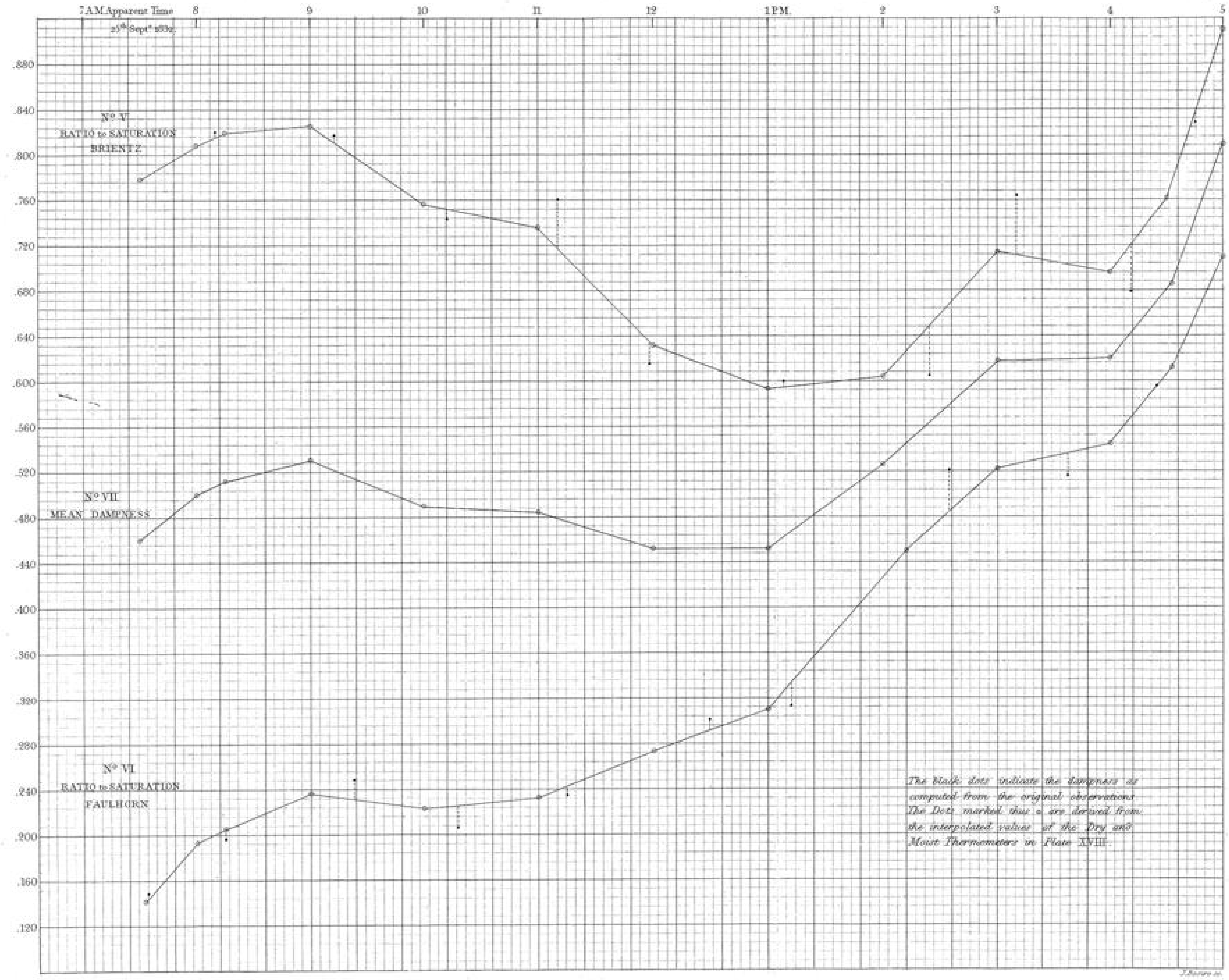
I have certainly not exaggerated here the difficulties in respect of weather. During the summer which has passed since the reading of this paper (1842), I have sedulously sought the opportunity of making some additional actinometer observations amongst the Alps, under the most favourable circumstances. But though the season was, as every one knows, more than commonly fine, I did not succeed in getting a single series of observations worth preserving, of the kind which I required. Excepting about three days in the end of June, and perhaps as many in the middle of August, the whole summer presented no unexceptionable weather, and on these two occasions I was unavoidably prevented from making use of my instruments, of which I had taken two from England on purpose. One experiment which I desired to make was, to push the observations to still greater thinness of atmosphere than could be obtained at the Faulhorn so late as the month of September, and for this purpose I proposed to ascend the Cramont (8966 feet) soon after the summer solstice, and I actually did spend a whole day without result on the summit in the month of July. By making single observations throughout the greater part of a day, I proposed to push the experimental Curve XV. further than had yet been done.

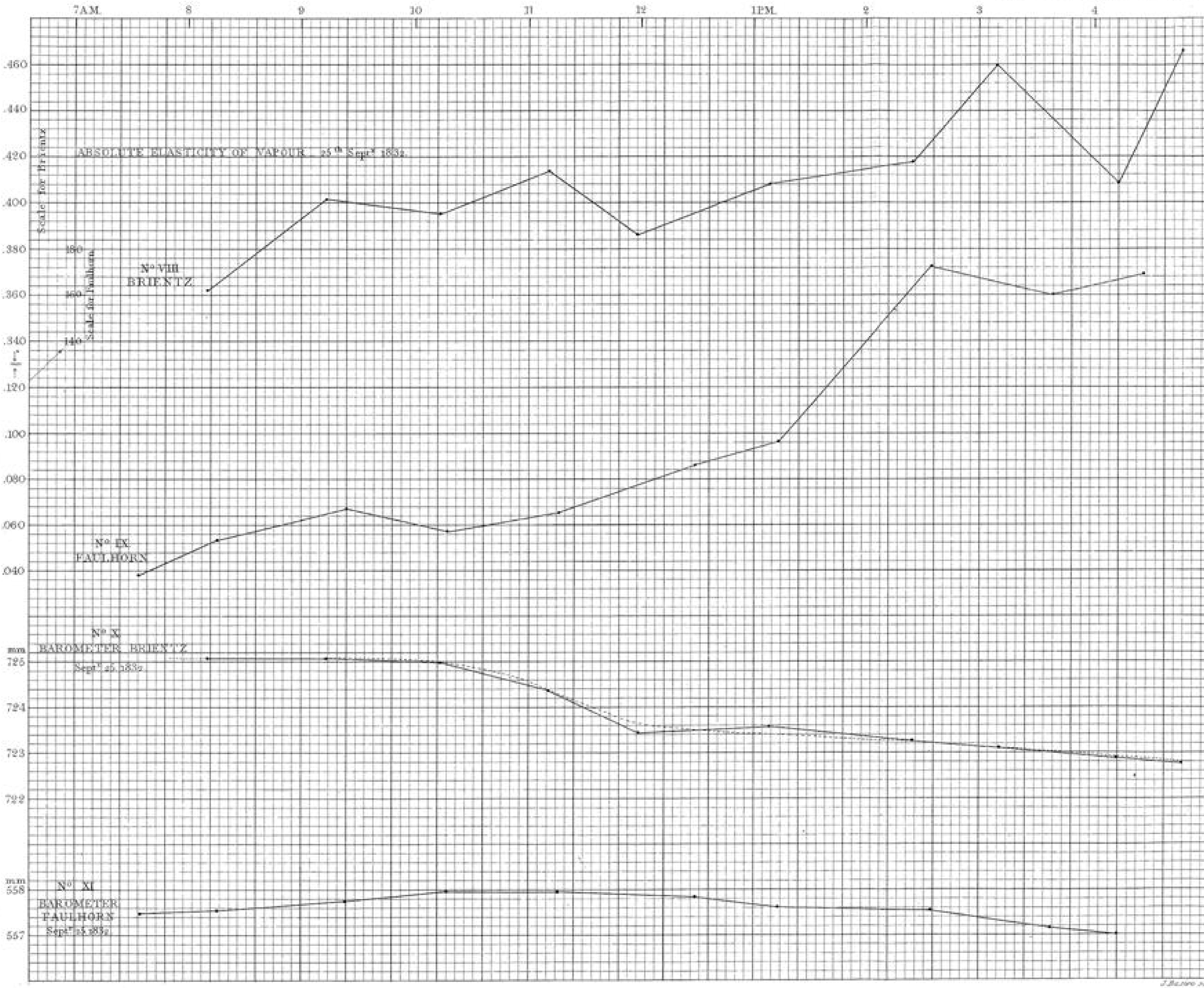
I also proposed another experiment which I recommend to future observers. I coated the bulb of one actinometer with white paint. The comparative value of the scale of this and another naked or dark blue bulb, would depend upon the nature of the incident heat (see Art. 3.). With heat from terrestrial sources transmitted by *most* diathermanous bodies, the *colour* of the surface becomes more and more influential, as the heat has been drained of its more absorbable part. But it would be very interesting to verify the fact in the case of the solar rays passing through air. For this purpose I proposed to compare the white and the dark actinometers at the top and at the bottom of a high mountain such as the Cramont, and I expected to find the disproportion least above and greatest below.



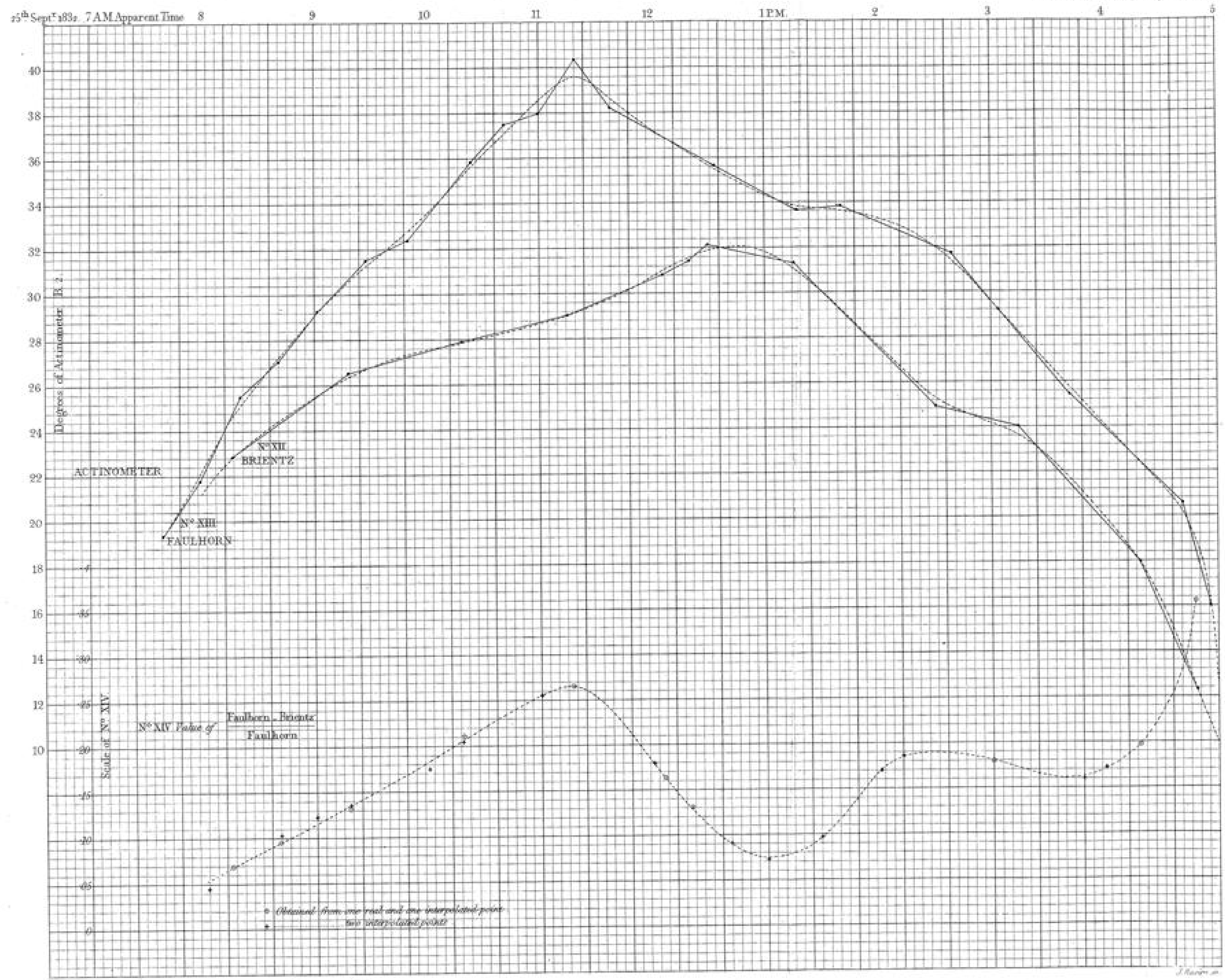
7 A.M. Apparent Time. 8
9
10
11
12
1 P.M.
2
3
4
5
Fah.
69°
67
65
63
61
59
57
55
46
44
42
40
38
36
34
32
30
28
26
24
Nº I.
THERMOMETER, BRIENTZ.
Nº III.
MOIST THERMOMETER
BRIENTZ.
Nº II. THERMOMETER
FAULHORN.
Nº IV.
MOIST THERR.
FAULHORN.
Scale for Dry & Moist Ther. Brientz.

J. Basire sc.


7 A.M. Apparent Time
25th Sept. 1832.
8
9
10
11
12
1 P.M.
2
3
4
5
.880
.840
.800
.760
.720
.680
.640
.600
.560
.520
.480
.440
.400
.360
.320
.280
.240
.200
.160
.120
No. V
RATIO to SATURATION
BRIENTZ
No. VII
MEAN DAMPNESS
No. VI
RATIO to SATURATION
FAULHORN
The black dots indicate the dampness as computed from the original observations.
The Dots marked thus ⊙ are derived from the interpolated values of the Dry and Moist Thermometers in Plate XVIII.
J. Basire sc.


7 A.M.
8
9
10
11
12
1 P.M.
2
3
4
5
ABSOLUTE ELASTICITY OF VAPOUR _ 25th Septr 1832.
Scale for Brientz
Scale for Faulhorn
.460
.440
.420
.400
.380
.360
.340
.120
.100
.080
.060
.040
180
160
140
No VIII
BRIENTZ
No IX
FAULHORN
No X
BAROMETER BRIENTZ
Septr 25. 1832
mm
725
724
723
722
No XI
BAROMETER
FAULHORN
Septr 25. 1832
mm
558
557

J. Basire sc.


25th Septr 1832. 7 A.M. Apparent Time
8
9
10
11
12
1 P.M.
2
3
4
5
Degrees of Actinometer B.2
Scale of Nº XIV
ACTINOMETER
Nº XIII
FAULHORN
Nº XII
BRIENTZ
Nº XIV Value of Faulhorn – Brientz / Faulhorn
Obtained from one real and one interpolated point
two interpolated points
40
38
36
34
32
30
28
26
24
22
20
18
16
14
12
10

N°. XV

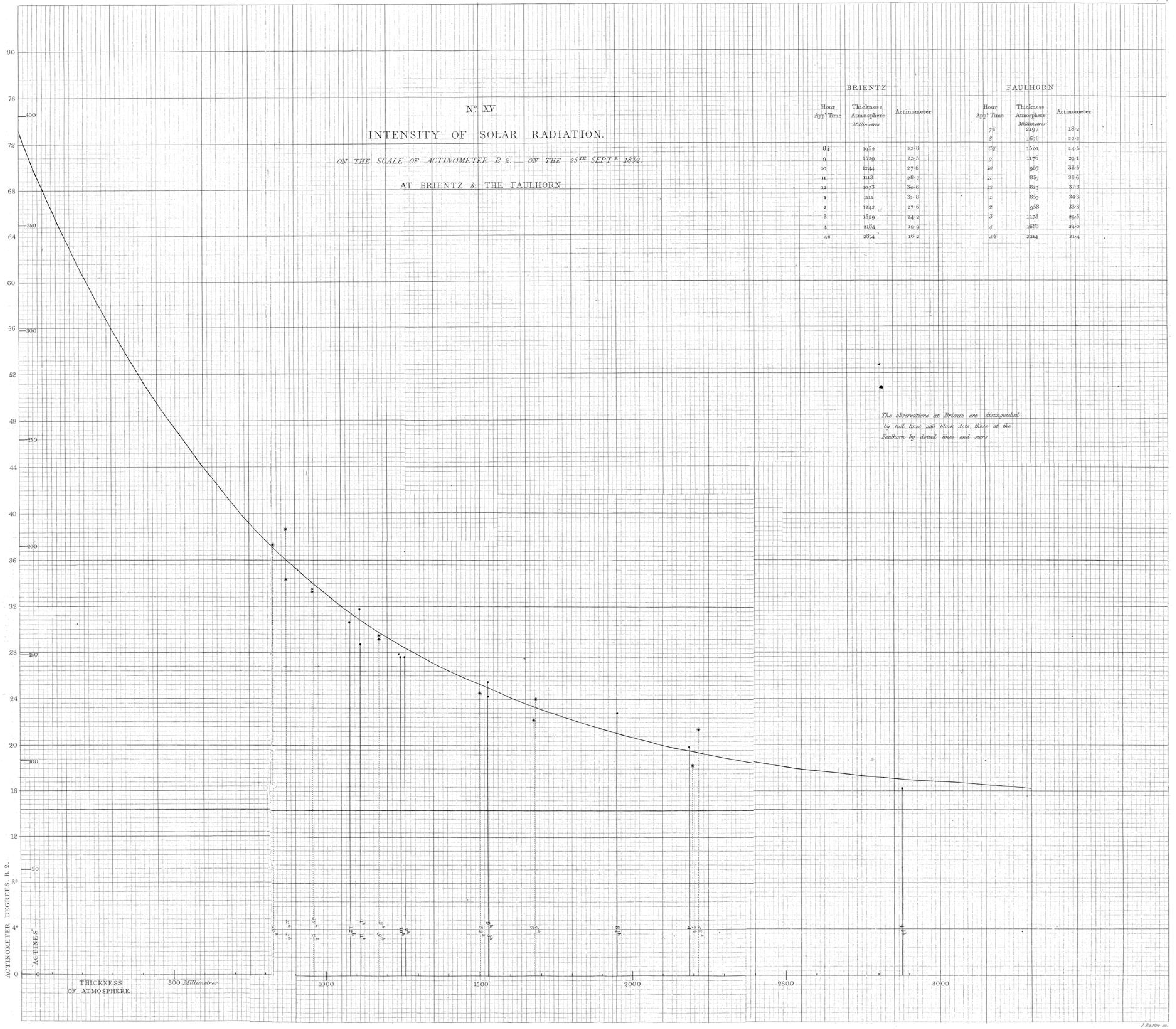
INTENSITY OF SOLAR RADIATION.

ON THE SCALE OF ACTINOMETER B. 2. — ON THE 25TH SEPTR. 1832.

AT BRIENTZ & THE FAULHORN.

BRIENTZ			FAULHORN		
Hour Appt. Time	Thickness Atmosphere *Millimetres*	Actinometer	Hour Appt. Time	Thickness Atmosphere *Millimetres*	Actinometer
			7¾	2197	18·2
			8	1676	22·2
8¼	1952	22·8	8¼	1501	24·5
9	1529	25·5	9	1176	29·1
10	1244	27·6	10	957	33·5
11	1113	28·7	11	857	38·6
12	1073	30·6	12	827	37·3
1	1111	31·8	1	857	34·3
2	1242	27·6	2	958	33·3
3	1529	24·2	3	1178	29·5
4	2184	19·9	4	1683	24·0
4¼	2874	16·2	4¼	2214	21·4

The observations at Brientz are distinguished by full lines and black dots, those at the Faulhorn by dotted lines and stars.



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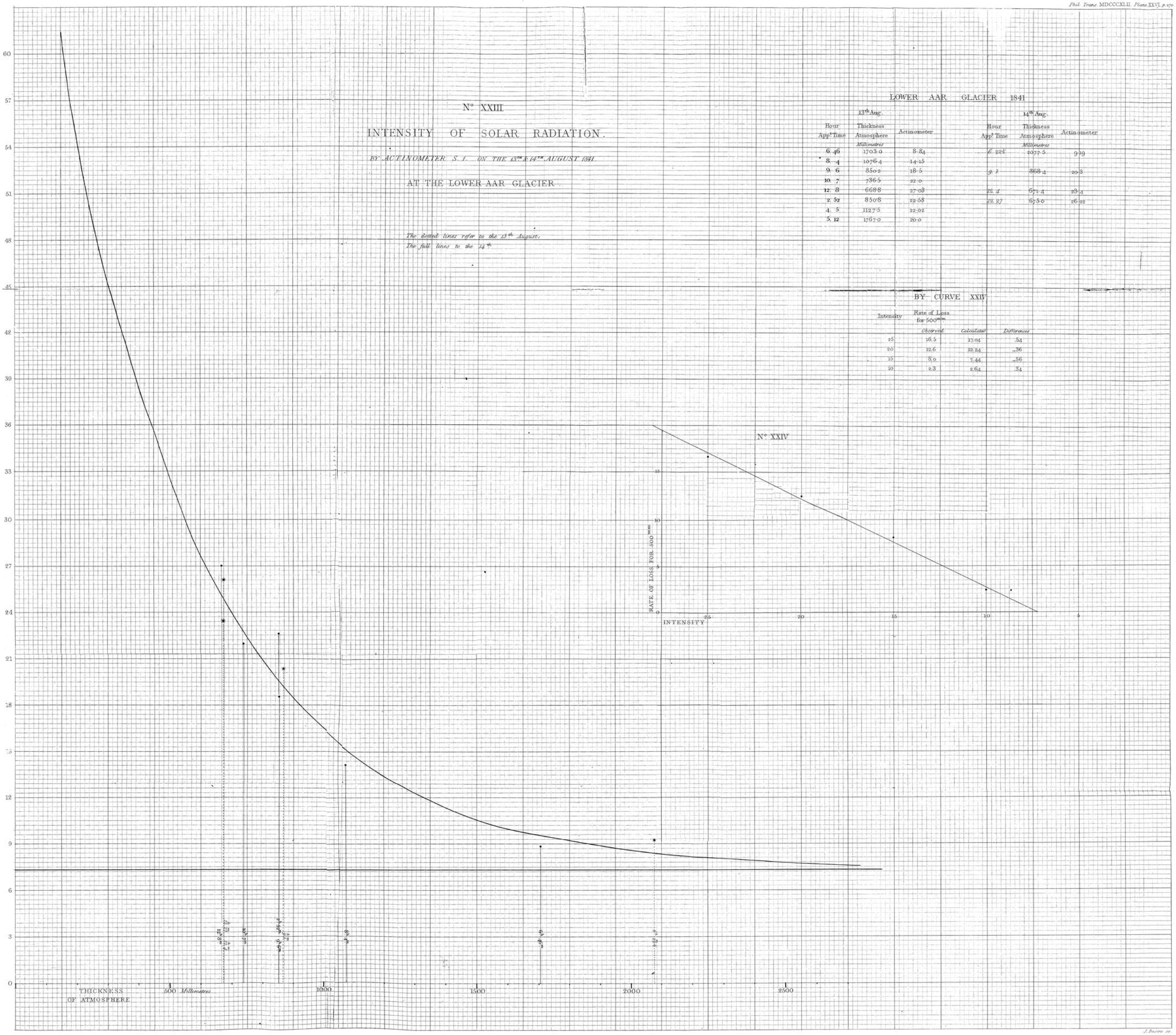

LOWER AAR GLACIER 1841

13th Aug. Hour Appt. Time	13th Aug. Thickness Atmosphere Millimetres	13th Aug. Actinometer	14th Aug. Hour Appt. Time	14th Aug. Thickness Atmosphere Millimetres	14th Aug. Actinometer
6. 46	1703·0	8·84	6. 22½	2077·5	9·19
8. 4	1076·4	14·15			
9. 6	850·2	18·5	9. 1	868·4	20·3
10. 7	736·5	22·0			
12. 8	668·8	27·03	12. 4	671·4	23·4
2. 52	850·8	22·58	12. 27	675·0	26·22
4. 5	1127·5	22·02			
5. 12	1767·0	20·0			

BY CURVE XXIV

Intensity	Rate of Loss for 500mm Observed	Calculated	Differences
25	16.5	17.04	.54
20	12.6	12.24	–.36
15	8.0	7.44	–.56
10	2.3	2.64	.34

J. Basire sc.